Optimal Redistribution with Unionized Labor Markets*

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Abstract

We analyze optimal redistributive taxation in an economy where labor markets are unionized and individual labor supply responds along the extensive (participation) margin. We show that the optimal tax and benefit system, in addition to the standard redistributive purpose, also serves to alleviate distortions induced by unions. In particular, income taxes should be lower the larger are the welfare gains associated to lowering involuntary unemployment. With unionized wage-setting, it may therefore be optimal to subsidize participation even for workers whose welfare weight is below one, something that can never be optimal when labor markets are competitive. Furthermore, we find that increasing the bargaining power of unions representing low-income workers is unambiguously welfare-enhancing, while the opposite holds true for high-income workers. Unions for low-skilled workers increase the effectiveness of the tax and benefit system to redistribute income, as any adverse impact from these unions on employment can be perfectly offset by lowering income taxes at the bottom. Hence, despite the fact that unions distort an efficient functioning of the labor market, allowing low-income workers to organize themselves is socially desirable for equity reasons.

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1 Introduction

This paper aims to answer two closely related questions concerning optimal income redistribution in unionized labor markets. The first question is: *How should the government optimize income redistribution when labor markets are unionized and labor supply responds along the extensive margin?* In other words, how should the government optimally design its tax and benefit system in an environment where individuals make a participation decision and labor market outcomes are determined through bargaining between unions and firms? The second question is: *Are unions a useful institution for redistribution?* Put differently, can it ever be desirable from a social welfare point of view to allow workers to organize themselves in a union?

The first question has, to the best of our knowledge, not yet been addressed in the literature. This is surprising given both its apparent empirical and policy relevance. Indeed, even though union membership rates have fallen drastically in recent decades in both Europe (Waddington, 2005) and the United States (Western and Rosenfeld, 2011), unions continue to play a predominant role in the determination of labor market outcomes, mostly notably in continental Europe. For instance, between 63% (Germany) and 99% (Austria) of wage and salary earners are directly covered or affected by collective union-negotiated agreements (Visser, 2006). Furthermore, the extensive (or participation) margin is often considered the empirically more relevant one when compared to the intensive (or hours) margin, especially at the lower part of the income distribution (see, for instance, Blundell and Macurdy, 1999, and Nichols and Rothstein, 2015). And while there are numerous studies that analyze the consequences of (income) taxation in unionized labor markets where individual labor supply is either exogenous (e.g. Palokangas, 1987, Bovenberg and van der Ploeg, 1994, Andersen et al., 1996, Koskela and Vilmunen, 1996, Boeters and Schneider, 1999, Koskela and Schöb, 2002) or concentrated along the intensive margin (e.g. Sørensen, 1999, Fuest and Huber, 2000, Aronsson and Sjögren, 2002, Aronsson et al., 2005, Kessing and Konrad, 2006, Aronsson et al., 2009, Koskela and Schöb, 2012), the extensive margin has thus far been ignored in this context. The first aim of this paper is to close this gap by analyzing how the government should design its tax and benefit system when labor markets are unionized and individual labor supply responses are concentrated along the extensive margin. In doing so, we merge the literature on optimal taxation with extensive labor supply responses pioneered by Diamond (1980) with the aforementioned studies that analyze the interaction between unions and (optimal) taxation.

The second question is motivated by the observation that the decline in unionization in recent decades has been accompanied by a sharp increase in inequality, as recently documented by Jaumotte and Buitron (2015). Other studies by Freeman (1980), Freeman (1991), Machin (1997), Lemieux (1998), Card (2001), Fairris (2003), Card et al. (2004), Checchi and Pagani (2004), DiNardo and Lemieux (1997), Koeniger et al. (2007), Visser and Checchi (2011), Mishel (2012) also find that stronger unions are associated to lower (wage) inequality. And while unions are generally considered to distort an *efficient* functioning of the labor market, the question arises whether unions could potentially be a useful institution for *equity* considerations. A highly similar question related to the desirability of a minimum wage has recently received renewed attention in the literature (see, e.g., Hungerbühler and Lehmann, 2009, Lee and Saez, 2012 and Gerritsen and Jacobs, 2014). In a similar fashion, this paper examines if it could ever be socially desirable to allow workers to organize themselves in a union and if so, under which conditions.

We will tackle these questions by analyzing an economy that includes workers, unions, firm-owners and a government. Workers are heterogeneous with respect to both their occupation (or type, or sector) and their costs of participation. Each type of labor is organized in a union whose goal it is to maximize the expected utility of its members. Firm-owners own a stock of capital and need labor of different types to produce the final consumption good. Equilibrium wages are determined through bargaining between firm-owners and unions. Importantly, we allow each union's bargaining power (or, equivalently, the degree of unionization) to vary across sectors. The latter ensures that the canonical model with purely extensive labor supply responses and competitively determined wages, analyzed in Saez (2002) and Christiansen (2015), is nested as a special case (in particular the case in which all unions have zero bargaining power). Furthermore, there is a government which can transfer resources between workers and firm-owners. The government can observe the employment status of all workers, as well as the occupation (or, equivalently, the wage) of each employed worker. The government can levy a fully nonlinear income tax. Participation costs are unobservable, so that the unemployment benefit is uniform. Finally, the government can observe and tax the income of firm-owners. The government chooses its tax instruments to maximize a utilitarian objective, subject to a budget constraint and the labor market equilibrium conditions. We characterize the policy optimum and examine how, once the government has optimally chosen its tax instruments, varying the bargaining power of the union in a particular sector affects social welfare.

Our main findings are the following. With respect to the first question, we derive an expression for the optimal participation tax (given by the sum of the income tax and the unemployment benefit) written solely in terms of estimable statistics and parameters related to the redistributive preferences of the government. We show that our optimal tax formula consists of both the conventional 'redistributive' component, as well as a novel 'corrective' component. The latter reflects the government's desire to alleviate distortions induced by unions. In particular, we show that income taxes (and hence participation taxes) should be lower the larger are the welfare gains associated to decreasing involuntary unemployment. Intuitively, by lowering income taxes, the government makes the employed workers better off relative to the unemployed. This motivates the unions (who also care about the well-being of their unemployed members) to moderate their wage claims, which leads to less involuntary unemployment. When the welfare gains associated to lowering involuntary unemployment are high, the government should exploit this channel and set income taxes at a low level. It is subsequently established that our expression for the optimal participation tax generalizes the result stated in Saez (2002) (for exogenous wages) and Christiansen (2015) (for the case with competitively determined wages)[1]. In stark contrast to the result obtained in these studies, we show that because of the corrective component, it may be optimal to subsidize participation on a net basis (i.e. setting an income subsidy -the negative of an income tax- that exceeds the unemployment benefit) even for workers whose welfare weight is below one.

With respect to the second question, our most important result is that increasing the bargaining power of unions representing low-income workers (i.e. the workers whose welfare weight exceeds one) is welfare-enhancing, while the opposite holds true for high-income workers. This result implies that, from a

[1] As will be explained in Section 4, the expressions for the optimal participation tax in these studies coincide, which is essentially an application of what is dubbed by Saez (2004) the 'Tax-Formula result', first derived in the seminal work of Diamond and Mirrlees (1971).

welfarist perspective, it is socially desirable to let low-skilled workers organize themselves in a union, whereas the wages for the more productive workers should preferably be determined competitively. Intuitively, the presence of unions for low-income workers allows the government to more efficiently use its tax and benefit system to redistribute income. By lowering income taxes at the bottom, the government can offset any adverse effects from stronger unions for low-skilled workers on employment. The decrease in the income taxes for low-income workers, in turn, can be financed by raising the taxes elsewhere in the economy, which has positive redistributional implications. A similar result is obtained in Lee and Saez (2012), who show that a minimum wage, despite -like unions- causing involuntary unemployment, may also be welfare-enhancing provided that the burden of involuntary unemployment is borne by the workers who experience lowest surplus from working (i.e. provided that rationing is efficient). In stark contrast, our finding regarding the desirability of unions continues to hold even when rationing is inefficient. This is because, unlike with minimum wages, under unionized wage-setting the government can use its tax and benefit system to influence the unions' wage claims, thereby affecting labor market outcomes.

The remainder of this paper is organized as follows. Section 2 discusses the related literature. Section 3 outlines the basic structure of the model and characterizes the general equilibrium for a given set of tax instruments. The question how these instruments should be optimally set is then addressed in Section 4. Section 5 subsequently examines how, once the government has optimally designed its tax and benefit system, changing the bargaining power of a particular union affects social welfare and moves on to characterize the welfare-maximizing degree of unionization in each sector. Section 6 investigates the robustness of the results by relaxing the assumption of efficient rationing. We analyze two extensions of the model in Section 7 and state some concluding remarks in Section 8.

2 Related Literature

Our paper relates to four branches in the literature. Firstly, our study contributes to the literature pioneered by Palokangas (1987) that studies optimal (redistributive) taxation under unionized wage-setting. Like the model we employ in the current paper, this study considers a setting with multiple types of labor and a utilitarian government. Because there are no informational frictions, the government can set its tax instruments to ensure that wages are uniform across sectors and the marginal utility of income is equated between all employed and unemployed workers, thereby achieving the first-best. Contrary to Palokangas (1987), the studies by Fuest and Huber (1997), Koskela and Schöb (2002), Aronsson and Sjögren (2004) and Aronsson and Wikström (2011) all consider an economy with only one type of labor (represented by a union), no informational frictions and show that also in this environment the government is able to achieve the first-best provided that profit taxation is unrestricted. Aronsson and Sjögren (2002), Aronsson et al. (2005), Kessing and Konrad (2006), Aronsson et al. (2009), in turn, consider an economy with more than one type of labor and unionized wage-setting, but instead of assuming that individual labor supply is exogenous (as in the above studies), they allow for intensive labor supply responses. As in the seminal work of Mirrlees (1971), the government can neither observe wages nor working hours. This informational distortion gives rise to self-selection constraints, which prevents the economy to attain the first-best. Our paper differs from these studies by considering optimal redistributive taxation with extensive, rather than intensive labor supply responses. As in the models with intensive responses, due to informational frictions the first-best is no longer attainable in our model.

Secondly, there is an extant literature that analyzes the impact of tax progressivity on employment in an environment with unionized wage-setting, as in Koskela and Vilmunen (1996), Fuest and Huber (1997), Sørensen (1999), Fuest and Huber (2000), Aronsson and Sjögren (2004), Sinko (2004), Aronsson and Wikström (2011), among others. When labor supply is concentrated along the intensive margin and wages are competitively determined, an increase in tax progressivity (typically modeled as an increase in the marginal tax rate, while keeping the average tax rate constant) reduces labor supply incentives, which leads to lower employment. With unionized wage-setting, however, this logic turns its head. Unions, when deciding upon their wage claim, face a trade-off between wages and employment. When tax systems become more progressive, a larger share of an increased wage claim is taxed away by the government, which shifts the union's trade-off in favor of employment. Consequently, under unionized wage-setting, increased tax progressivity is actually good for employment. A key aspect in the above studies, however, is that individual labor supply is exogenous. If workers can individually respond by changing the number of hours they work or the effort they put into their job, the impact of increased progressivity on overall employment (defined as the total number of hours worked) becomes ambiguous (Fuest and Huber, 2000, Koskela and Schöb, 2012). As we will ignore any intensive margin considerations and because we let the government directly choose the level of taxation at each point in the wage distribution (which in the above models would correspond to picking an average tax rate, while setting the marginal tax rate equal to zero), our model has little, if anything, to say about the desirability of tax progressivity per se. Nevertheless, in line with the above literature, we will show that considerations related to the impact of the tax and benefit system on (un)employment play a crucial role in the derivation of the optimal tax formulae.

Thirdly, this paper contributes to the literature on optimal redistributive taxation with extensive labor supply responses, pioneered by Diamond (1980). In stark contrast to the optimal income tax schedule with purely intensive labor supply responses (first derived in the seminal work of Mirrlees, 1971), Diamond (1980) shows that the optimal transfer program involves subsidizing participation of low-income workers (i.e. those groups of workers whose welfare weight exceeds one). In order to do so, the optimal transfer program combines negative marginal tax rates at the bottom with a small guaranteed income, much in the spirit of an Earned Income Tax Credit (EITC) program. For a more recent and very general treatment of optimal taxation with extensive labor supply responses, we refer to Choné and Laroque (2011). While Diamond (1980) and Choné and Laroque (2011) consider an economy with a continuum of wages (or equivalently, productivities), in the models of Saez (2002) and Christiansen (2015) the number of labor types is finite and our exposition is more similar to theirs. However, in contrast to the aforementioned studies where wages are either exogenous (as in Diamond, 1980, Saez, 2002 and Choné and Laroque, 2011) or competitively determined (as in Christiansen, 2015), we employ a general production function and assume union-negotiated wages. Since we allow for a varying degree of the unions' bargaining power and because our production function admits both perfect and imperfect substitutability among different types of labor, our expression for the optimal participation tax nests the one depicted in the studies mentioned above as a special case. The result that subsidization of low-income workers is optimal continues to hold in our model, but we additionally show that with unions, it may even be optimal to subsidize participation for workers whose welfare weight is below one.

Finally, our study relates closely to a recently emerging literature that studies the optimality of minimum wage policies in conjunction with optimal income taxation. Like unions, a binding minimum wage increases the income of certain groups of workers at the costs of creating involuntary unemployment. Lee and Saez (2012) show that introducing a minimum wage for low-income workers is welfare-enhancing, provided that the burden of involuntary unemployment is borne by the workers who experience the lowest surplus from working (i.e. provided that rationing is efficient). Importantly, this argument holds true even if the government has optimized its tax and benefit system. Somewhat paradoxically, this is because a minimum wage, unlike income taxes and unemployment benefits, can create involuntary unemployment. However, when the assumption of efficient rationing is relaxed, Gerritsen and Jacobs (2014) show that a minimum wage generally ceases to be optimal. In stark contrast, we will show that our result regarding the desirability of unions is unaffected by any concerns related to the specifics of the rationing scheme. The reason is that, unlike with a minimum wage, with unions the government can still use its tax instruments to affect labor market outcomes by influencing the unions' wage claims. This mechanism enables the government to fully offset any adverse effects from stronger unions on employment, something that can never be achieved by means of the tax and benefit system once a binding minimum wage is introduced. Like Lee and Saez (2012) and Gerritsen and Jacobs (2014), Hungerbühler and Lehmann (2009) also analyze the optimality of a minimum wage, but do so in an economy with matching frictions where wages are determined through bargaining between individual workers (*not* organized in a union) and firms. They show that introducing a minimum wage may be optimal if the bargaining power of the workers is 'too low' (i.e. below the level required for the Hosios condition to be satisfied). If, on the other hand, the government could also increase the workers' bargaining power, it would be socially desirable to do so and introducing a minimum wage ceases to be optimal. Similarly to this result, we also demonstrate that increasing the bargaining power of workers (more precisely, the bargaining power of unions) may improve social welfare. However, in stark contrast to the model from Hungerbühler and Lehmann (2009) where the bargaining power of individual workers is uniform across productivity types, we allow the bargaining power of the unions to vary across sectors and show that only an increase in the bargaining power of unions representing low-skilled workers improves social welfare.

3 Model

Throughout we consider an economy that includes workers, unions, firm-owners and a government. Workers are heterogeneous with respect to their occupation and their costs of participation. Within each occupation, workers are represented by a union and labor market outcomes are determined through collective bargaining between unions and firm-owners. The latter own a stock of capital and operate a technology that is used to produce the final consumption good, which is sold in a perfectly competitive market at a price normalized to one. The government can use its tax instruments to transfer resources between unemployed workers, employed workers and firm-owners. As is standard in the public finance literature (see, e.g. Diamond, 1980, Saez, 2002, Choné and Laroque, 2011, Christiansen, 2015), we assume that the government can observe the occupation (or type) only for the individuals that are actually employed. Hence, income taxes can be conditioned on wages (which vary across occupations), but the unemployment benefit is uniform. Individual-specific participation costs are unobservable so that none of the taxes can be conditioned on this characteristic. Finally, the government can also observe and tax the income of the firm-owners, which consists of the firm's profits.

3.1 Workers

There is a discrete number of I types of labor (to which we will refer to as occupations, or sectors) and we denote by N_i the mass of workers of type $i \in \mathcal{I} \equiv \{1, .., I\}$. Each worker is endowed with one indivisible unit of time that can be used in the production process of the final consumption good. Whenever a worker supplies this unit of time to a firm (i.e. whenever the worker is employed), he or she incurs so-called participation costs. These costs are denoted by φ and vary across workers. In particular, within an occupation i the distribution of participation costs is described by a cumulative distribution function $G_i(\varphi)$ with support $[\underline{\varphi}_i, \overline{\varphi}_i]$, where $\underline{\varphi}_i < \overline{\varphi}_i \leq \infty$. For analytical convenience, we assume that participation costs are of the pecuniary type, so that they directly translate into a reduction in income and hence consumption (see Choné and Laroque, 2011). The preferences of the workers are then defined over *net* consumption (i.e. the workers' income minus any potential costs of participation) and described by a utility function $u(\cdot)$ that satisfies the standard properties $u'(\cdot), -u''(\cdot) > 0$.

We denote by w_i the wage earned by the workers who are employed in occupation i and by T_i the corresponding income tax these workers have to pay. Unemployed workers, in turn, receive an unemployment benefit equal to $-T_u$ units of consumption (so that T_u denotes the tax 'paid' by the unemployed). Because we always assume that participation is voluntary, then if an individual in occupation i with participation costs φ is employed, it must be that

$$v_i(\varphi) \equiv u(w_i - T_i - \varphi) \geq u(-T_u) \equiv v_u. \tag{1}$$

Hence, whenever an individual is employed, this implies that he or she prefers being employed over being unemployed. The reverse, however, need not be true. In particular, if for some unemployed workers (1) is satisfied, this simply means that these workers are *involuntarily* unemployed.

3.2 Unions

Workers in sector i are organized in a union, whose goal it is to maximize the expected utility of its members. For simplicity, we assume that each type i worker, independent of his or her costs of participation, is a member of the union. Equivalently, we may say that the fraction of type i workers who are union-members is representative for the population of type i workers. Needless to say, this assumption is hard to defend on empirical grounds. However, as long as the union cares about the well-being of the members who face a positive probability of becoming involuntarily unemployed, the qualitative predictions of the model remain robust to the specifics of the union's objective.

Workers within each occupation differ along two dimensions: their individual-specific participation costs and their employment status. Therefore, in order to determine the expected utility of workers in occupation i, we need to take a stance on how unemployment is allocated among workers with different participation costs. That is, we have to say something about the *rationing scheme*. For now, we will make the following assumption:

Assumption 1. *Efficient rationing: the workers who remain unemployed are the ones with the highest costs of participation.*

When labor markets are competitive, all unemployment is voluntary and Assumption 1 is trivially satisfied. If, however, part of the unemployment is involuntary (which may happen when labor markets are unionized), then there is no reason to believe that only individuals with the highest participation costs will bear the burden of unemployment, unless there would be a secondary market for jobs (Gerritsen, 2013). Assumption 1 thus fails to recognize that unions can also induce distortions regarding the *incidence* of unemployment and will therefore be relaxed in Section 6.

If we denote by E_i the employment rate in occupation i, then Assumption 1 implies that individuals with participation costs $\varphi \in [\underline{\varphi}_i, G_i^{-1}(E_i)]$ are employed, whereas those with participation costs $\varphi \in (G_i^{-1}(E_i), \overline{\varphi}_i]$ remain unemployed. In words, whenever the employment rate is E_i and rationing is efficient, this means that the workers with the lowest participation costs become employed first, up and to the point where the fraction of participants equals E_i. This happens at the point where participation costs are equal to $G_i^{-1}(E_i)$. The expected utility of workers in sector i can then be written as

$$\Lambda_i = \int_{\underline{\varphi}_i}^{G_i^{-1}(E_i)} u(w_i - T_i - \varphi) dG_i(\varphi) + \int_{G_i^{-1}(E_i)}^{\overline{\varphi}_i} u(-T_u) dG_i(\varphi) = E_i \overline{v_i} + (1 - E_i) v_u, \tag{2}$$

where $\overline{v_i} \equiv \mathbb{E}[v_i(\varphi) | \underline{\varphi}_i \leq \varphi \leq G_i^{-1}(E_i)]$ denotes the expected, or average utility of the workers who are employed in sector i.

3.3 Firm-owners

There is a normalized mass of one firm-owners who own K units of capital and operate a technology $F(K, L_1, .., L_I)$ that is used to produce the final consumption good, where $L_i \equiv E_i N_i$ denotes the amount of type i labor that is used in the production process. We assume that $F(\cdot)$ features constant returns to scale and satisfies the following conditions

$$F_K(\cdot), F_i(\cdot) > 0, \qquad F_{KK}(\cdot), F_{ii}(\cdot) \leq 0, \qquad F_{Ki}(\cdot), F_{ij}(\cdot) \geq 0, \tag{3}$$

for all $i \neq j$. The subscripts denote the partial derivatives with respect to capital and type i (j) labor. The conditions in (3) state that production is increasing in all its inputs at a non-increasing rate and that all inputs are (at least weakly) cooperative in production. Firm-owners do not incur any costs of participation and simply consume the firm's profits net of taxes. If we denote by T_f the tax paid by firm-owners, then their well-being can be written as

$$v_f \equiv u(F(K, L_1, .., L_I) - \sum_i w_i L_i - T_f). \tag{4}$$

Firm-owners choose how much labor to hire in order to maximize (4). The first-order conditions are given by the familiar

$$w_i = F_i(K, L_1, .., L_I) \tag{5}$$

for all i. At this point, it is important to emphasize that individual firm-owners, when deciding how much laborers to hire, take wages as given. The equilibrium wages, in turn, are determined through bargaining between unions and (representatives of) firm-owners. Thus, given the wage that is agreed upon in the bargain, individual firms have 'consumer sovereignty' when it comes to the amount of labor they wish to hire. The specifics of the bargaining procedure are discussed in detail when we characterize the general equilibrium of the economy under consideration. Before doing so, however, we first need to introduce the final actor in this economy: the government.

3.4 Government

There is a government which can transfer resources between firm-owners and workers. The latter differ in terms of their employment status, their occupation and their participation costs. It is assumed that the government can observe the employment status of all workers. In addition, the government can observe the pretax incomes (or, equivalently, the occupations[2]) of the workers who are employed. Individual participation costs, however, are unobservable. A direct consequence is that the government cannot observe whether unemployed workers are voluntarily or involuntarily unemployed. Finally, the government can identify the firm-owners and observe the profits made by the firms.

In line with the above information structure, the income tax T_i is allowed to vary across occupations, which implies that the government can levy a fully nonlinear income tax. The unemployment transfer, on the other hand, is uniform as the government can neither observe an unemployed worker's type nor his or her participation costs. Furthermore, the government can tax the income of firm-owners. Now, if we denote by R the exogenous (and possibly negative) revenue raised by the government, the budget constraint reads

$$R + \sum_i N_i(E_i T_i + (1 - E_i)T_u) + T_f = 0. \tag{6}$$

Turning to the government's objective, we assume that the government has utilitarian preferences. This means that the government aims to maximize the unweighted sum of all agents' utilities. Using (2) and (4), our measure of social welfare is given by

$$\mathcal{W} = \sum_i N_i(E_i \overline{v_i} + (1 - E_i)v_u) + v_f. \tag{7}$$

It is important to emphasize that, by the concavity of the agents' (common) utility function $u(\cdot)$, a utilitarian government prefers a more equal distribution of income over a very unequal one. And while it is straightforward to allow for a more general specification of social welfare, as long as the government values the well-being of *all* agents, the qualitative predictions of the model remain robust to such a modeling choice. The only commonly employed welfare function for which this may not be the case is the one used by a Rawlsian government. With Rawlsian preferences, the government only cares about the well-being of the individuals in the economy that are worst off and consequently attaches zero weight to the well-being of individuals from all other categories. We will investigate this case separately in Section 7.2.

3.5 Equilibrium

Now that the main actors in our economy are identified, we turn to characterize the general equilibrium of the economy under consideration. Importantly, we do so in two steps. Firstly, the remainder of this section characterizes the (private-sector) equilibrium for a given level of taxation. Then, in Section 4 we turn to address the question how the government should optimally set its tax instruments, fully taking

[2] We ignore the possibility that workers of different occupations may earn the same pretax wage. Since workers in different occupations generally vary in terms of their marginal productivities, the distribution of the participation costs and the bargaining power of the union representing them (to be discussed later), there is no reason to believe that pretax wages will be the same.

into account how taxes affect the private-sector outcomes. By choosing this approach, we implicitly assume that the government is the Stackelberg leader relative to all actors in the private sector, including the unions. We will comment on the implications of this modeling choice in more detail in Section 8.

To characterize the general equilibrium for a given level of taxation, the only feature that remains to be specified is how the (partial) equilibrium in the market for each type of labor is determined. Throughout we assume that the wage in sector i is set in a bargain between the union representing type i workers and (representatives of) firm-owners. Individual firm-owners, in turn, take wages as given and decide how much labor to hire according to (5). Importantly, we assume that bargaining takes place at the sectoral level and that all unions bargain with the firm-owners independently from each other. Hence, unions do not coordinate their actions. Furthermore, we allow the bargaining power of the union (or the degree of unionization) relative to that of the firm-owners to vary across sectors. This description of the labor market corresponds to what is known in the literature as the Right-to-Manage model (see, for instance, Heijdra, 2009). This model owes its name to the fact that, while wages are determined at the aggregate (sectoral) level, individual firms have the 'right to manage' how much labor they hire. A well-known feature of this model is that it nests both the competitive equilibrium, as well as the Monopoly Union outcome (due to Dunlop, 1950) as a special case, each for a specific degree of the union's bargaining power. We will discuss these special cases in turn and then characterize how the equilibrium is determined for any degree of unionization.

If a union in sector i has full bargaining power (which is the case in the Monopoly Union model), it is going to choose the wage, or equivalently the rate of employment, that maximizes its objective (2) subject to the firm's labor demand curve (5). As stated, in doing so, each union takes the actions of the other unions as given. Since maximizing with respect to the rate of employment is slightly more convenient than maximizing with respect to the wage, we characterize the equilibrium in the Monopoly Union by solving

$$\max_{E_i} \left\{ \int_{\underline{\varphi}_i}^{G_i^{-1}(E_i)} u(F_i - T_i - \varphi) dG_i(\varphi) + \int_{G_i^{-1}(E_i)}^{\overline{\varphi}_i} u(-T_u) dG_i(\varphi) \right\}, \tag{8}$$

where the firm's optimality condition $w_i = F_i$ is substituted out for in the union's objective. For analytical convenience, we assume that the union's objective is strictly concave in E_i (after $w_i = F_i$ is substituted out for), so that the first-order condition is both necessary and sufficient. The latter, in turn, reads:

$$\int_{\underline{\varphi}_i}^{G_i^{-1}(E_i)} u'(w_i - T_i - \varphi) dG_i(\varphi) F_{ii} + u(w_i - T_i - G_i^{-1}(E_i)) - u(-T_u) = 0 \quad \Leftrightarrow$$

$$E_i \overline{v_i'} + E_i'(v_{i,m} - v_u) = 0. \tag{9}$$

Here, $\overline{v_i'}$ denotes the average marginal utility of the employed in sector i and $E_i' = 1/F_{ii}$ is the slope of the labor demand curve[3]. In addition, we define by

$$v_{i,m} \equiv u(w_i - T_i - G_i^{-1}(E_i)) \tag{10}$$

[3]For now and in the remainder, whenever we write F_i or F_{ij} (for any j), we use the convention that these terms are evaluated in the point $(K/N_i, L_1/N_i, .., E_i, .., L_I/N_i)$ unless explicitly stated otherwise. When differentiating with respect to E_i, this turns out to be slightly more convenient.

the well-being of the *marginally* employed, or marginal worker in sector i. This is the worker with the highest participation costs that is still employed. The union's first-order condition has a very clear interpretation. It states that in the optimum, the benefit of marginally increasing the wage, which leads to a higher well-being of the employed (reflected by the term $E_i\overline{v_i'}$), equals the marginal cost of increasing the wage claim. The latter consists of the decrease in the rate of employment multiplied by the utility loss of the marginally employed workers and is captured by the term $E_i'(v_{i,m} - v_u)$. The reason why the loss in employment is weighed by the utility loss of the marginal worker directly follows from our assumption of efficient rationing: if there is a decrease in employment following an increase in the wage, by Assumption 1 it will be the employed workers with the highest participation costs, i.e. the marginal workers, who lose their jobs first. What is furthermore noteworthy to point out, is that from the union's first-order condition it can be inferred that if there would be a decrease in either the unemployment benefit or the income tax, the union would respond optimally by decreasing the wage claim. Intuitively, both an increase in T_u and a decrease in T_i make the employed workers better off relative to the unemployed. This will motivate the union, who cares about the expected utility of its members, to moderate the wage claim, which in turn leads to a higher rate of employment.

In the opposite case where the union has no bargaining power at all, the partial equilibrium in the market for type i labor in the Right-to-Manage model coincides with the outcome that would occur when labor markets are competitive. In this case, competition among workers within the same occupation ensures that the wage is driven to the point where the marginally employed worker is indifferent between participating and not participating, i.e.

$$v_{i,m} = v_u \quad \Leftrightarrow \quad E_i = G_i(w_i - T_i + T_u), \tag{11}$$

using (10). The above relationship gives the labor supply curve, as it denotes the fraction of workers that are willing to participate when the wage equals w_i, for a given tax and benefit system. The competitive outcome can then be found by combining the labor supply and labor demand curve (5).

For any intermediate degree of the union's bargaining power, we can characterize the equilibrium in the market for type i labor as follows. First, let us define by $\rho_i \in [0,1]$ the bargaining power of the union representing type i workers or, equivalently, the degree of unionization in sector i. Now, any labor market equilibrium in the Right-to-Manage model can be written as a solution to the following equation:

$$\rho_i E_i\overline{v_i'} + E_i'(v_{i,m} - v_u) = 0. \tag{12}$$

Whenever $\rho_i = 0$, the solution to (12) coincides with the competitive outcome, as given by (11) (when combined with the labor demand schedule $w_i = F_i$). If, on the other hand, $\rho_i = 1$, then the outcome in the Right-to-Manage model coincides with the monopoly outcome, as given by (9) (where the firm's optimality condition was already used in the union's optimization problem). Since the equilibrium employment rate (wage) can readily be verified to be decreasing (increasing) in the union's bargaining power, it directly follows that any equilibrium in the Right-to-Manage model corresponds to a particular choice of $\rho_i \in [0,1]$. The higher (lower) ρ_i, the closer will the outcome lie to the monopoly (competitive) outcome.

The above discussion is graphically illustrated in Figure 1. The point *CE* denotes the competitive

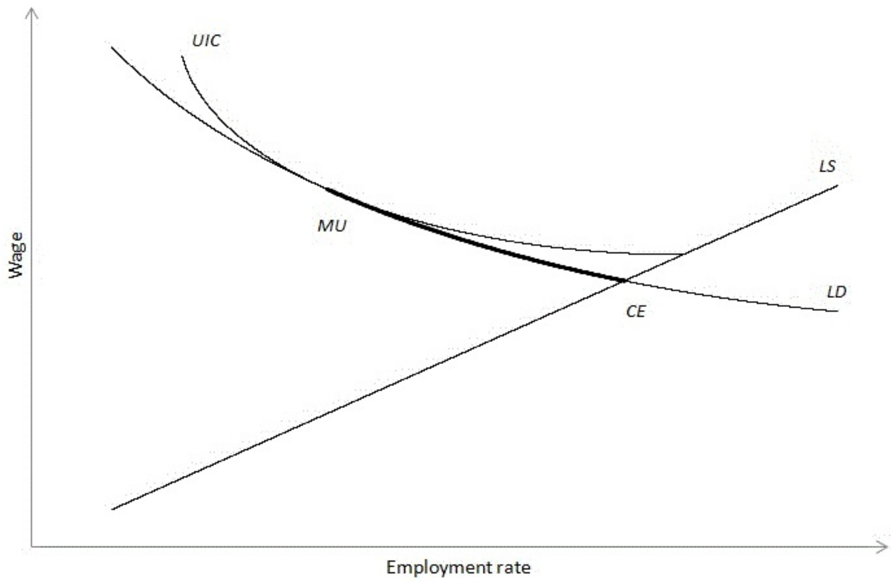

Figure 1: Labor market equilibria in the Right-to-Manage model

equilibrium and lies at the intersection of the labor supply (LS) curve and the labor demand (LD) curve, which correspond to (11) and (5) respectively. The point MU denotes the outcome that would occur if a monopoly union could unilaterally choose the combination of wages and employment on the labor demand curve that maximizes its objective (2). Naturally, this outcome lies at the point where the union's indifference curve (UIC) is tangent to the labor demand curve. Now, every point on the bold part of the labor demand curve corresponds to an equilibrium for a particular choice of the union's bargaining power ρ_i. The higher (lower) ρ_i, the closer the equilibrium lies to the point MU (CE).

Equation (12) and Figure 1 highlight an important observation: for any nonzero degree of the union's bargaining power ρ_i, the equilibrium in the labor market for sector i will always feature some degree of *involuntary* unemployment. To see why this is true, note that (12) implies that whenever $\rho_i > 0$, it must be that $v_{i,m} > v_u$. In words, the marginally employed worker is strictly better off than the unemployed workers. This implies that there are also unemployed individuals who would actually prefer to be employed rather than remaining unemployed. Graphically, the presence of involuntary unemployment is depicted as follows. Whenever $\rho_i > 0$, the equilibrium lies strictly to the left of the labor supply curve, so that given the wage, more individuals would like to work than are hired by the firm. The reason why the presence of unions always results in some involuntary unemployment is the following. Starting from the competitive equilibrium, marginally raising the wage increases the well-being of all employed workers. From the union's perspective, this constitutes a first-order welfare gain. The associated decrease in the rate of employment, however, is not accompanied by a first-order welfare loss. Indeed, under the assumption of efficient rationing, the individuals that enter unemployment first are the ones that are indifferent between employment and unemployment. Consequently, whenever rationing is efficient, the union always prefers an outcome with a higher wage and some degree of involuntary unemployment over an equilibrium without involuntary unemployment. This implies that, whenever unions have nonzero bargaining power, there will always be unemployed workers who would actually prefer to be employed in equilibrium.

Having determined how the (partial) equilibrium in each labor market is determined for a given degree of the union's bargaining power, we are now ready to characterize the general equilibrium. For a given level of taxation, the equilibrium employment rates in all sectors can be found by solving the following system of equations

$$\rho_i \int_{\underline{\varphi}_i}^{G_i^{-1}(E_i)} u'(F_i - T_i - \varphi) dG_i(\varphi) F_{ii} + u(F_i - T_i - G_i^{-1}(E_i)) - u(-T_u) = 0, \tag{13}$$

for all i. The above constitutes a system of I equations in I unknowns, i.e. the equilibrium employment rates, and simply corresponds to solving (12) for all i. As stated before, given the equilibrium employment rates, the equilibrium wages then directly follow from the labor demand curves (5).

For future purposes, it is useful to emphasize that the system of equations given by (13) determines the equilibrium employment rate in each sector i as a function of the income taxes paid by *all* the workers (i.e. the solution for E_i generally depends on all the T_i's and T_u). This follows from the notion that the marginal productivity of type i labor (and hence, the labor demand schedule) typically depends on the amount of labor that is employed in other sectors as well. Indeed, only when $F_{ij} = 0$ for all $i \neq j$ will the equilibrium rate of employment in sector i depend solely on T_i and T_u and not be affected by the income taxes in any of the other sectors. In the remainder, we will refer to this special case as one where labor markets are *independent*, since in this case a change in the employment rate in one sector does not affect the labor market outcomes in any of the other sectors. An example of a production function that gives rise to independent labor markets (and which satisfies the conditions stated in (3)) is

$$F(K, L_1, ..., L_I) = a_k K^\alpha \left(\sum_i a_i L_i^{1-\alpha} \right), \tag{14}$$

where $\alpha \in (0,1)$ and $a_k, a_1, ..a_I > 0$. Note that, since all labor types are complementary to capital, the production function in (14) does *not* correspond to a technology in which labor types are perfect substitutes, despite the fact that $F_{ij} = 0$ for all $i \neq j$. The latter would require that marginal productivities are constant (i.e. $F_{ii} = 0$ for all i), which is not satisfied for the specification in (14).

4 Optimal Taxation

Now that we have characterized the general equilibrium for a given level of taxation, we turn to address the question how the government should optimally set its tax instruments. The latter consist of income taxes T_i, an unemployment tax T_u (or, equivalently, a benefit $-T_u$) and a tax T_f paid by the firm-owners. The government chooses the combination of taxes and labor market outcomes that maximizes its objective (7), subject to the budget constraint (6), the firm's optimality conditions (5) and the labor market equilibrium conditions (13). After using the firm's optimality conditions to substitute out for the equilibrium wages, we can write the government's problem as

$$\begin{aligned}
\max_{T_1,...,T_I,T_u,T_f,E_1,...,E_I} \mathcal{L} &= \sum_i N_i \left(\int_{\underline{\varphi}_i}^{G_i^{-1}(E_i)} u(F_i - T_i - \varphi) dG_i(\varphi) + \int_{G_i^{-1}(E_i)}^{\overline{\varphi}_i} u(-T_u) dG_i(\varphi) \right) \\
&+ u(F - \sum_i F_i N_i E_i - T_f) + \lambda(R + \sum_i N_i(E_i T_i + (1 - E_i) T_u) + T_f) \\
&+ \sum_i \mu_i \left(\rho_i \int_{\underline{\varphi}_i}^{G_i^{-1}(E_i)} u'(F_i - T_i - \varphi) dG_i(\varphi) F_{ii} + u(F_i - T_i - G_i^{-1}(E_i)) - u(-T_u) \right),
\end{aligned} \tag{15}$$

where λ is the shadow value of an additional unit of resources R and the μ_i's denote the multipliers on the labor market equilibrium conditions. Allowing the government to directly choose the employment rates in addition to the tax instruments is validated by the observation that the labor market equilibrium conditions stated in (13) implicitly define the equilibrium employment rates as a function of all the taxes paid by the (un)employed. Alternatively, we could write these relationships as $E_i = E_i(T_1, .., T_I, T_u)$, use the resulting expressions to substitute out for the employment rates in the government's objective and budget constraint and only then optimize with respect to the taxes. In the remainder, we will also pursue this approach. For now, however, letting the government directly choose the employment rates while explicitly taking into account the labor market equilibrium conditions turns out to be both more convenient and insightful.

For future references, it is useful to introduce the social welfare weights of the different groups of agents in the economy. In particular, let

$$b_i = \frac{\overline{v_i'}}{\lambda}, \quad b_u = \frac{v_u'}{\lambda}, \quad b_f = \frac{v_f'}{\lambda} \tag{16}$$

denote the welfare weight of the employed workers in sector i, the unemployed workers and the firm-owners, respectively. In words, the welfare weight of a particular group measures how much the government values an increase in the consumption of the individuals belonging to this group relative to how much it values receiving an additional unit of resources R. Using (16), we can straightforwardly manipulate the first-order conditions with respect to the taxes (see Appendix A for details) to obtain the results summarized in the next Proposition.

Proposition 1. *Consider the economy as described in Section 3. In the policy optimum,*

(i) the welfare weight of firm-owners equals one, i.e.

$$b_f = 1, \tag{17}$$

(ii) a weighted average of the employed and unemployed workers' welfare weights equals one. More specifically,

$$\sum_i \omega_i b_i + \omega_u b_u = 1, \tag{18}$$

where the weights ω_i and ω_u (which are strictly positive and sum to one) are given by

$$\omega_i \equiv \frac{N_i E_i v_u'}{\rho_i E_i \overline{v_i''} F_{ii} + v_{i,m}'} \left(\sum_j N_j \left[\frac{E_j v_u'}{\rho_j E_j \overline{v_j''} F_{jj} + v_{j,m}'} + (1 - E_j) \right] \right)^{-1} \tag{19}$$

$$\omega_u \equiv \sum_i N_i (1 - E_i) \left(\sum_j N_j \left[\frac{E_j v_u'}{\rho_j E_j \overline{v_j''} F_{jj} + v_{j,m}'} + (1 - E_j) \right] \right)^{-1}. \tag{20}$$

Proof. See Appendix A. □

Point (i) of Proposition 1 states that it is optimal for the government to tax the income of firm-owners up to the point where the latter's welfare weight equals one. This directly follows from the fact that the profit tax is fully non-distortive, as it does not affect any of the agents' decisions. At the margin, the government should therefore be indifferent between receiving an additional unit of revenue R and raising the firm-owners' consumption by one unit.

The second insight from Proposition 1 is that also a weighted average of the welfare weights of the different groups of workers must be equal to one in the policy optimum. As will be made clear in the next few paragraphs, this result implies that whenever the government want to raise one unit of revenue from the workers, it should do this in such a way that the welfare costs of raising this unit of revenue (which consists of the decrease in well-being of the workers) equals the government's valuation of this additional unit of resources. Both results stated in Proposition 1 essentially confirm the claim postulated by Jacobs (2013) that in the presence of redistributional concerns, the marginal cost of public funds (defined as the value of a unit of resources raised by the government relative to the value of that unit of resources to the private sector) must be equal to one in the policy optimum, irrespective of whether taxes are distortionary or not.

To grasp some intuition for the weights attached to the different groups of workers, it is insightful to start by considering the setting where labor markets are competitive (i.e. $\rho_i = 0$ for all i). In that case, it can readily be established that the expression for the weights simplify to (see Appendix A for details)

$$\omega_i = \frac{N_i E_i}{\sum_j N_j} \tag{21}$$

$$\omega_u = \frac{\sum_i N_i (1 - E_i)}{\sum_j N_j}. \tag{22}$$

Hence, with competitive labor markets the weight attached to a particular group simply equals that group's share in the total labor force. This result goes back to Saez (2002) and implies that whenever the government has optimally set its tax instrument and aims to raise one additional unit of resources from the workers, the best thing it can do is to increase the taxes of all workers (employed *and* unemployed) by *exactly* the same amount. In the policy optimum, both the direct welfare costs (i.e. the decrease in the workers' well-being) and the welfare gain (i.e. the relative value of an additional unit of government resources) associated to this policy intervention are equal to one. Essential in bringing about this result is that an increase in T_i and T_u that leaves $T_i - T_u$ unaffected will not influence any of the participation decisions by the workers. Consequently, whenever labor markets are competitive, the aforementioned policy intervention does not have any distortive effects.

If the degree of unionization is nonzero in at least one sector, then the weights from (19)-(20) differ from the shares in the labor force. This, in turn, implies that if the government wants to raise one unit of resources from the workers, it is generally no longer optimal to increase all income taxes (including the tax paid by the unemployed) by exactly the same amount. The reason is that, in stark contrast to the case with competitive labor markets, under unionized wage-setting such a policy intervention (i.e. an increase in T_i and T_u that leaves $T_i - T_u$ unchanged for all i) will *not* leave the labor market outcomes unaffected. This readily follows from the observation that, unlike the individual's participation decision, the union's optimality condition is not symmetrically affected by T_i and T_u (see (9)). Consequently, there is no reason to believe that the aforementioned policy intervention leaves the wage claim of the union, and hence the labor market outcomes, unaffected. And it is exactly for this reason that the direct link between the weights from (19)-(20) and the population shares breaks down whenever the degree of unionization is nonzero in at least one sector.

It is furthermore noteworthy to emphasize that point (ii) from Proposition 1 implies that the welfare weight of some groups of workers will exceed one, while for others their welfare weight is below one. To see why this is true, note that voluntary participation ensures that employed workers are always better of than the unemployed, which by (16) implies that $b_u > b_i$ for all i. Then, from (18), it must be that there is at least one group of workers for whom $b_i < 1$. Obviously, unless $I = 1$, this does not imply that the welfare weight of *all* groups of employed workers should be below one. Indeed, because there may be large differences between workers from different sectors, there will generally also be groups of employed workers whose welfare weight is above one (for instance the working poor). Without further notice, we will refer to these workers as low-income, or low-skilled workers. Workers whose welfare weight is below one (which, by (17), roughly corresponds to those workers who, in the policy optimum, are on average better off than the firm-owners) will then be referred to as high-income, or high-skilled workers.

Combined, the results from Proposition 1 state that the government should redistribute from firm-owners to workers (or vice versa) in such a way that the firm-owners' welfare weight equals a weighted average of the welfare weights of the different groups of workers. And while this finding may give an idea of what proportion of the firm's profits the government should tax away in the policy optimum, it provides little guidance when it comes to the question how the government should optimally set its income taxes. To that end, consider the following Proposition.

Proposition 2. *Consider the economy as described in Section 3. In the policy optimum, the tax schedule must satisfy*

$$N_i E_i (1 - b_i) + \sum_j N_j E_j (b_j - 1) \frac{\partial w_j}{\partial T_i} + \sum_j N_j \left[\frac{1}{\lambda}(v_{j,m} - v_u) + (T_j - T_u) \right] \frac{\partial E_j}{\partial T_i} = 0, f \tag{23}$$

where $\partial w_j / \partial T_i$ and $\partial E_j / \partial T_i$ measure the impact of a change in the income tax in sector i on the equilibrium wage and employment rate in sector j. Furthermore, whenever labor markets are independent (so that $F_{ij} = 0$ for all $i \neq j$), (23) simplifies to

$$T_i - T_u = -\frac{1}{\lambda}(v_{i,m} - v_u) + \left(\frac{w_i}{\varepsilon_{wi}} - \frac{T_i}{\varepsilon_{Ti}} \right)(1 - b_i), \tag{24}$$

where ε_{wi} denotes the elasticity of the demand for type i labor with respect to the wage w_i and ε_{Ti} denotes the elasticity of the equilibrium employment rate in sector i with respect to the income tax T_i. These elasticities, in turn, are given by

$$\varepsilon_{wi} = \frac{1}{F_{ii}} \frac{w_i}{E_i} < 0 \tag{25}$$

$$\varepsilon_{Ti} = \frac{\rho_i E_i \overline{v_i''} F_{ii} + v'_{i,m}}{\rho_i E_i \overline{v_i''} F_{ii} F_{ii} + \rho_i E_i \overline{v_i'} F_{iii} + v'_{i,m} F_{ii} + \rho_i v'_{i,m} F_{ii} - v'_{i,m}/G'_i} \frac{T_i}{E_i} < 0. \tag{26}$$

Proof. See Appendix B. $\square$

To understand the economics behind the results stated in Proposition 2, first consider (23). Suppose that the government is interested in the welfare consequences of marginally increasing the income tax T_i. This policy intervention will have both a mechanical, as well as behavioral effects. Concerning the first, the increase in T_i raises the resources available to the government at the expense of lowering the consumption of the employed workers in sector i. Since the government's relative valuation of its own resources is one and that of the employed workers in sector i is b_i, the welfare effect of this change in the income tax is proportional to $1 - b_i$. This effect is captured by the first term in (23).

Now, let us turn to discuss the behavioral consequences associated to marginally increasing T_i. Following the increase in the income tax, the union representing the workers in sector i will be motivated to increase the wage claim. Intuitively, if a change in the tax system makes the employed workers worse off, the union wants to compensate these workers by demanding a higher wage. And since a change in the wage in sector i will directly affect the employment rate in sector i, which affects the marginal productivity and hence the demand for other types of labor as well (provided that labor markets are not independent), a change in T_i potentially influences the labor market outcomes in all sectors. The welfare effects associated to these changes are captured by the second and third term in (23) respectively. Starting with the second term, note that a change in the wage in sector j following a change in T_i effectively constitutes a change in the transfer from the firm-owners (whose welfare weight is one) to employed workers in sector j (whose welfare weight is b_j). Therefore, the changes in the equilibrium wages are weighed by $b_j - 1$. Concerning the third term, there are two welfare effects associated to a change in the employment rate in sector j. Firstly, the individuals that will find (lose) employment in sector j following an increase in the income tax in sector i experience a utility gain (loss) equal to $v_{j,m} - v_u$ (which should be divided by λ to translate the change in utils into consumption units). Secondly, the movement of a type j worker from unemployment to employment will cause a change in the government's budget equal to $T_j - T_u$. What the result in (23) thus states is that all these welfare effects should be balanced at the optimum, so that marginally increasing the income tax has a net welfare effect of zero.

While relatively straightforward to interpret, the expression from (23) seems particularly hard to work with empirically, especially because it requires knowledge of how a change in the income tax in one sector affects wages and employment rates in other sectors. Whenever these cross-effects are absent (or, more likely, thought to be of second-order importance) then the result from (23) simplifies considerably and we obtain the relationship given by (24). The latter is written solely in terms of estimable statistics and parameters related to the (redistributive) preferences of the government and does not require any knowledge of the aforementioned cross-effects. Therefore, this expression is much easier to work with and could potentially be used to either evaluate a tax system that is in place or could be helpful to characterize the optimal tax and benefit system. We will now discuss this relationship in more detail.

The left-hand side of (24) measures the change in the taxes paid by an individual when he or she moves from unemployment to employment. Therefore, this term is commonly referred to in the literature as the *participation tax*. In the optimum, the participation tax must equal the sum of two components. The first term reflects the government's desire to alleviate distortions induced by unions, whereas the second term reflects the government's desire to redistribute income towards workers with a high social welfare weight. Starting with the first, the term $\frac{1}{\lambda}(v_{i,m} - v_u)$ measures the (monetized) increase in social welfare associated to a decrease in involuntary unemployment in sector i. As such, it is a measure of the distortive impact of the union in sector i. Whenever labor markets are competitive, the marginally employed worker will be indifferent between employment and unemployment (i.e. $v_{i,m} = v_u$) and this term would simply cancel. If, however, the degree of unionization is positive and the equilibrium features involuntary unemployment, then the result in Proposition 2 states that the income tax (and hence the participation tax) should be lower, the larger is the utility loss the marginally employed worker experiences whenever he or she loses his or her job. And since the term $v_{i,m} - v_u$ is higher the stronger is the union, the result directly implies that the income tax should *ceteris paribus* be lower in those sectors where unions cause larger distortions. To understand the mechanisms behind this finding, recall that a decrease in the income tax puts downward pressure on the union's wage claim. Whenever the corresponding increase in employment results in a large (direct) welfare gain for the individuals that enter employment (i.e. $v_{i,m} - v_u$ is large), then it is optimal for the government to exploit this channel by setting a low income tax.

Turning to the second term, the result depicted in (24) highlights that the income tax for a particular group of workers should be lower the higher is their welfare weight[4]. As stated, this reflects the government's desire to redistribute income towards workers with a high social welfare weight. The extent to which an increase in the welfare weight of a particular group of workers translates into a decrease in the income tax, in turn, is governed by two elasticities: the elasticity of labor demand and the elasticity of the equilibrium employment rate with respect to the income tax. To understand the role of the labor demand elasticity, suppose that an increase in the income tax has a strong positive impact on the equilibrium wage. This will motivate the government to set a high income tax only if the associated redistribution from firm-owners to workers is welfare-enhancing (which happens if and only if $b_i > 1$). The impact on the equilibrium wage is less likely to be strong when the labor demand schedule is nearly horizontal, i.e. the more elastic is labor demand. In addition, the role of the elasticity of the equilibrium employment rate with respect to the income tax ε_{Ti} is identical to the role played by $\partial E_i / \partial T_i$ in (23) and which we discussed before: when weighing the costs and benefits of increasing the income tax, the government should take into account how the income tax affects the rate of employment.

As a final remark, it is worthwhile to point out that the result stated in (24) implies that in the case of independent labor markets, it is always optimal to subsidize participation on a net basis (i.e. setting $T_i - T_u < 0$) for low-income workers. That is, workers whose welfare weight exceeds one should optimally receive an income subsidy that exceeds the unemployment benefit. This finding goes back to Diamond (1980) and is also derived in Saez (2002). However, in stark contrast to the finding from these studies, upon inspecting (24) it should be clear that the reverse need not be true whenever wage-setting is unionized. In other words, when labor markets are unionized, it may be optimal to subsidize participation on a net basis even for workers whose welfare weight is below one. From (24) it can be verified that this may happen whenever the term $\frac{1}{\lambda}(v_{i,m} - v_u)$ is sufficiently positive. As explained in the above, this term captures the distortive impact of unions. And while there are no direct redistributional gains from subsidizing high-income workers, recall that income taxes (or subsidies) are also used to alleviate the distortions induced by unions. Thus, whenever this channel is sufficiently strong, it may be socially desirable to set the income tax at such a low level (rather, the income subsidy at such a high level) that *de facto* even the participation decision for workers whose welfare weight falls short of one is subsidized on a net basis in the policy optimum.

[4]In Appendix B, it is verified that the term that is multiplied by $1 - b_i$ is indeed positive.

4.1 Relation to Saez (2002)

In a seminal contribution, Saez (2002), among other things, derives in a straightforward fashion how the government should optimally set its participation taxes in a setting where labor supply is concentrated along the extensive margin and, importantly, wages are exogenously given. Under these assumptions, it is shown that the following relationship must hold in the policy optimum[5]:

$$T_i - T_u = \frac{w_i - T_i + T_u}{\eta_i}(1 - b_i), \tag{27}$$

where

$$\eta_i \equiv \frac{\partial G_i(w_i - T_i + T_u)}{\partial (w_i - T_i + T_u)} \frac{w_i - T_i + T_u}{G_i} \tag{28}$$

denotes the participation elasticity. The latter measures the percentage increase in the fraction of individuals in occupation i that are willing to participate if the difference in the consumption bundles between an employed and an unemployed worker (given by $w_i - T_i + T_u$) increases by one percent. For the details of the derivation and an intuitive explanation of the result, we refer to Saez (2002). Here, we only highlight that a direct implication of the result highlighted in (27) is that subsidizing participation on a net basis in sector i is optimal if and only if the welfare weight of workers exceeds one. As explained before, this sharply contrasts the finding that with unionized labor markets, it may also be optimal to subsidize participation for workers whose welfare weight is below one.

Now, since our expression for the optimal participation tax is derived in a setting where wages are endogenously determined in unionized labor markets (for any possible degree of unionization), it should come as no surprise that the result stated in Proposition 2 nests the Saez (2002) result as a special case. The following Corollary identifies the conditions under which both results coincide.

Corollary 1. *The result stated in Proposition 2 coincides with the result from Saez (2002) (who considers wages to be exogenously given) if either*

(i) wages are endogenously determined in unionized labor markets, provided that labor types are perfect substitutes in production (i.e. provided that labor demand is infinitely elastic),

(ii) wages are endogenously determined, provided that labor markets are perfectly competitive (for an arbitrary elasticity of labor demand),

or both.

Proof. See Appendix B. □

Firstly, Corollary 1 states that the expression for the optimal participation tax derived in Saez (2002) (with exogenous wages) also holds in an environment with unionized wage-setting, provided that labor demand is perfectly elastic. Intuitively, whenever labor demand is infinitely elastic, a marginal increase in the wage leads to a complete breakdown of employment. Therefore, a union would never demand a wage that is above the competitive level. It directly follows that whenever labor types are perfect substitutes (so that labor demand curves are horizontal), wages are essentially exogenous and the result from Saez (2002) readily carries over, irrespective of the degree of unionization.

[5] See Proposition 1 in Saez (2002).

The second insight from Corollary 1 is that the Saez (2002) result also holds in a setting where wages are endogenously determined, provided that labor markets are perfectly competitive (which, in our model, corresponds to the case where $\rho_i = 0$ for all i). And since the only behavioral elasticity that shows up in (27) is the participation elasticity, a direct implication of the equivalence of the optimal tax formulae is that labor demand considerations are irrelevant for the characterization of optimal participation taxes whenever labor markets are perfectly competitive. This finding is also very recently presented in Christiansen (2015), but actually goes back to the seminal work of Diamond and Mirrlees (1971). These authors show that the optimal tax formulae that are derived with exogenous prices readily carry over to a setting where prices are competitively determined. This result is labeled by Saez (2004) the 'Tax-Formula result' and the finding presented in (ii) is essentially an application of this result.

The above discussion makes clear that, unless labor markets are perfectly competitive and/or all labor types are perfectly substitutable, the result from Saez (2002) generally breaks down. Most importantly, we find that the expressions for the optimal participation tax should also reflect the government's desire to alleviate the distortions induced by unions. As emphasized above, this may provide a rationale to subsidize participation on a net basis even for workers whose welfare weight is below one, something that is never optimal when labor markets are competitive. Secondly, we show that labor demand considerations are no longer irrelevant for the characterization of the optimal participation taxes whenever the degree of unionization is nonzero (as can be seen from comparing (27) with (24)). This is consistent with the finding from Jacquet et al. (2012), who also derive an expression for the optimal participation tax in a framework with labor market imperfections (in particular, matching frictions). In line with the result stated in Proposition 2, these authors too identify a crucial role for the elasticity of labor demand in the expression for the optimal participation tax.

5 Desirability of Unions

The results from the previous section illustrate how the government should optimally design its tax and benefit system whenever labor markets are unionized. In this section, we take the analysis a step further by analyzing how, once the government has optimally set its tax instruments, an increase in the degree of unionization in a particular sector affects social welfare. By doing so, we aim to answer the question whether it can be socially desirable to allow workers to organize themselves in a union, and if so, under which conditions.

As was explained in the first section, there is considerable empirical evidence that the decline in unionization over the recent decades has been accompanied by a strong increase in (wage) inequality (see, e.g., Jaumotte and Buitron, 2015). Nevertheless, this finding in and of itself does not imply that unions are a desirable institution for redistributive purposes. As illustrated above, the presence of unions could lead to potentially severe efficiency losses. When unions bargain for a wage that is above the market-clearing level, this will ultimately result in involuntary unemployment. The accompanying welfare costs should therefore be weighed against the redistributional benefits (if any) that unions could potentially bring about. This question appears to be ultimately an empirical one. Nevertheless, since our measure of social welfare $\mathcal{W}$ fully reflects both the potential redistributional gains as well as efficiency costs that result from the presence of unions, we can use the theoretical framework constructed above to (i) shed light on the question whether it can ever be the case that unions are a desirable institution for redistribution and, if the answer is affirmative, (ii) identify conditions under which this is the case. The following Proposition addresses both these points.

Proposition 3. *Consider the economy as described in Section 3 and consider the case where labor demand is not perfectly elastic. In the policy optimum, increasing the bargaining power of the union in sector i is accompanied by an increase social welfare if and only if the welfare weight of the employed workers in sector i exceeds one. That is, whenever $F_{ii} < 0$,*

$$\frac{\partial \mathcal{W}}{\partial \rho_i} > 0 \quad \Leftrightarrow \quad b_i > 1. \tag{29}$$

Proof. See Appendix C. □

Proposition 3 states that, once the government has optimally set its tax instruments, increasing the bargaining power for low-income (high-income) workers is accompanied by an increase (a decrease) in social welfare, provided that labor demand curves are not horizontal. A direct implication of this result is that, despite the fact that unions distort an efficient functioning of the labor market, it is socially desirable on equity grounds to let low-income workers organize themselves in a union.

To understand the economics behind Proposition 3, it is instructive to start by considering the case where labor markets are independent and competitive (i.e. F_{ij} = for all $i \neq j$ and $\rho_i = 0$) and ask ourselves what happens when we introduce a union in a low-income sector (i.e. when we marginally increase ρ_i above zero in a sector where $b_i > 1$). If labor demand is not perfectly elastic, the introduction of a union will lead to an increase in the wage and an accompanying decrease in the rate of employment (see Figure 1). The rise in the equilibrium wage constitutes an increase in the transfer from firm-owners (whose welfare weight is one) to employed workers in sector i (whose welfare weight is above one). From the government's perspective, the welfare effect associated to this redistribution of income is positive. Furthermore, recall from our previous discussion that, in the policy optimum, participation is always subsidized on a net basis for low-skilled workers. The decrease in the rate of employment therefore positively affects the government's budget, which again has beneficial welfare implications. Furthermore, because of our assumption of efficient rationing, the employed workers who enter unemployment following the introduction of the union are the ones who are indifferent between being employed and unemployed. Hence, their movement into unemployment has a net welfare effect of zero. Adding up, it is immediately clear that introducing a union in a low-income sector unambiguously increases social welfare.

The above argumentation illustrates how introducing a union in a low-income sector affects social welfare when labor markets are independent and initially competitive. Another interpretation of the result stated in Proposition 3 (and one which holds true irregardless of the initial structure of the labor market) goes along the following lines. Suppose that there is an exogenous increase in ρ_i and that the government aims to use its tax instruments to ensure that the labor market outcomes remain unaffected. In order to do so, it needs to decrease the income tax T_i. This will motivate the union, whose bargaining power has just increased, to moderate the wage claim, which prevents the equilibrium wage in occupation i from rising. The decrease in the income tax for this particular group of workers can be financed, for instance, by increasing the profit tax[6]. The welfare effect associated to this policy intervention is proportional to $b_i - 1$, since the welfare weight of the workers whose income tax is decreased equals b_i and that of the firm-owners equals one. Hence, if the associated welfare impact of this policy intervention is positive, it is immediately implied that an increase the bargaining power of the union in a low-income sector positively affects social welfare.

What the above discussion makes clear is that unions increase the scope for the government to redistribute income towards the workers who are represented by a union, provided that labor demand is not infinitely elastic. Put differently, the combination of taxes and unions potentially leads to more efficient redistribution than what can be achieved by the combination of taxes and competitive labor markets. And, importantly, this can be achieved without necessarily harming employment. Indeed, the combined increase in the union's bargaining power and the policy intervention described in the previous paragraph brings about solely a transfer of income from firm-owners to low-income workers, leaving all labor market outcomes (i.e. all wages and (un)employment rates) unaffected. However, it should be noted that while the rate of unemployment in sector i remains unchanged, this is not true for the *nature* of unemployment. In particular, because of the increase in the after-tax wage of low-income workers (recall that the pretax wage is kept constant, while the income tax is lowered), an increased fraction of the unemployed workers are now *involuntarily* so, as opposed to having chosen not to participate. From a welfarist perspective, however, this distinction is irrelevant: the question whether an individual is voluntarily or involuntarily unemployed has no differential impact on this agent's contribution to social welfare.

5.1 The optimal degree of unionization

The analysis conducted above also allows us to take a first pass at addressing the question what the socially desirable degree of unionization is. That is, suppose that the government could use its political power to affect the relative strengths of the negotiating parties, how then would it choose to set the bargaining power of each union relative to that of the firm-owners? Obviously, a thorough analysis of this question requires a careful examination of the extent to which, and at what costs, the government is actually able to affect the unions' bargaining power. Indeed, in a similar fashion Hungerbühler and Lehmann (2009), in their study regarding the desirability of a minimum wage in a model with matching frictions, note that "Whether and how the government can affect the bargaining power is still an open question." (p.475). And whereas they consider an environment where wages are determined through bargaining between *individual workers* and firms, the authors nevertheless argue that changing the way in which *unions* are financed and regulated may be a way for the government to affect the workers' bargaining power. For now, we leave these issues aside and act as if the government can use its political power to costlessly dictate the bargaining power of each union. How it should optimally do so is then summarized in the next Corollary.

Corollary 2. *Consider the economy as described in Section 3 and consider the case where labor demand is not perfectly elastic. If the government could simultaneously determine the optimal degree of*

[6] It should be stressed that increasing the profit tax is only one of the many ways to finance the decrease in the income tax for workers in sector i. As long as the welfare costs of raising one unit of revenue are equal to one (which is always the case in the policy optimum; see Jacobs, 2013), the argumentation readily carries over.

unionization in each sector (in addition to the level of taxation), it would set

(i) $\rho_i = \min[\rho_i^*, 1]$ *whenever* $b_i \geq 1$,

(ii) $\rho_i = \max[\rho_i^*, 0]$ *whenever* $b_i \leq 1$,

where ρ_i^* *is the bargaining power of the union required to make the social welfare weight of the employed workers in sector* i *equal to one.*

Proof. See Appendix C. □

Point (i) from Corollary 2 states that for workers whose welfare weight exceeds one (i.e. the low-skilled workers), the government prefers to continue increasing the bargaining power of the union representing them until their welfare weight is equal to one. If the latter is not feasible (which may happen for workers who start out with a very low pretax income), then the best thing to do is to simply give the union representing these workers full bargaining power, i.e. to set $\rho_i = 1$. The opposite holds true for high-skilled workers. For these individuals, it is optimal for the government to lower their union's bargaining power, but it can never decrease the degree of unionization below the point $\rho_i = 0$.

It is noteworthy to point out that there is a clear similarity between the result stated in Corollary 2 and the result obtained in Lee and Saez (2012) in their study on the desirability of a minimum wage[7]. They show that, whenever rationing is efficient and labor demand is not perfectly elastic (the same conditions that apply in Corollary 2), increasing the minimum wage for low-income workers is welfare-enhancing until the point where the welfare weight of the low-income workers is equal to one. The intuition is very similar to the mechanism described above. The minimum wage effectively allows the government to redistribute more towards low-skilled workers, which should be done until the point where the latter's welfare weight is equal to one.

A final noteworthy observation is that Proposition 1 and Corollary 2 jointly imply that, from a social welfare point of view, there should always be at least one sector where unions are absent. To see why this is true, note that voluntary participation and our definitions of the social welfare weights imply that $b_u > b_i$ for all i. Then, from the second point of Proposition 1, it must be that there is at least one i for which $b_i < 1$. From Corollary 2 it is clear that unions should ideally be absent in this sector. An implication of this observation is that, when labor is homogeneous (i.e. $I = 1$), increasing the bargaining power of the only union that is present in the economy is always accompanied by a decrease in social welfare. Thus, only in an economy with multiple types of labor that are not perfectly substitutable in the production process[8] can it ever be the case that unions have the potential to increase social welfare beyond the point that can be achieved with competitive labor markets.

6 Inefficient Rationing

Throughout we have assumed that, whenever involuntary unemployment occurs in equilibrium, the burden of unemployment is always borne by the workers who experience the lowest surplus from working.

[7]See Proposition 2 in Lee and Saez (2012)

[8]The reason why labor types should not be perfectly substitutable, is that otherwise labor demand curves are horizontal and unions have no impact on any of the labor market outcomes (so that increasing the degree of unionization can never lead to an increase in social welfare).

In other words, rationing was assumed to be efficient. Now, one may be inclined to think that this assumption biases our results from the preceding section in favor of unions. This is indeed the case for the Lee and Saez (2012) result regarding the desirability of a minimum wage. In particular, the finding that introducing a minimum wage for low-skilled workers is welfare-enhancing whenever rationing is efficient does not readily carry over to a setting where the assumption of efficient rationing is relaxed. For instance, Lee and Saez (2008) show that introducing a minimum wage generally reduces welfare when rationing is uniform (which corresponds to the case where all employed workers, independent of their participation costs, are equally likely to lose their jobs). Gerritsen and Jacobs (2014) also show that the desirability of a minimum wage depends critically on the incidence of unemployment and argue that a minimum wage can always be 'made' optimal by making the appropriate assumptions on the rationing scheme. To investigate whether the claim that introducing unions for low-income workers increases social welfare (as stated in Proposition 3) also hinges crucially on the assumption of efficient rationing, we will now partly repeat the analysis from the preceding sections using a general rationing scheme.

We model the rationing scheme in a highly similar fashion as in Gerritsen (2013) and Gerritsen and Jacobs (2014)[9]. In particular, we specify a function $e_i(E_i, \varphi) \in [0, 1]$ that denotes the fraction of type i workers with participation costs φ that are employed when the 'aggregate' employment rate among type i workers is E_i. Then, by definition, the following relationship must hold for all E_i:

$$\int_{\underline{\varphi}_i}^{\overline{\varphi}_i} e_i(E_i, \varphi) dG_i(\varphi) = E_i. \tag{30}$$

For analytical convenience, we assume that for all values of φ, the function $e_i(E_i, \varphi)$ is differentiable with respect to its first-argument whenever $E_i < G_i(w_i - T_i + T_u)$ (i.e. whenever there is some involuntary unemployment[10]) and we denote this derivative by $e_i'(E_i, \varphi)$. For reasons to be made clear below, it is furthermore assumed that in this case $e_i'(E_i, \varphi) \geq 0$. In words, whenever there is an increase in the aggregate employment rate in sector i, the fraction of employed individuals with participation costs φ will not decrease.

With a general rationing scheme, the expected utility of type i workers (which by assumption corresponds to the union's objective) is given by

$$\Lambda_i = \int_{\underline{\varphi}_i}^{\overline{\varphi}_i} \left(e_i(E_i, \varphi) u(w_i - T_i - \varphi) + (1 - e_i(E_i, \varphi)) u(-T_u)\right) dG_i(\varphi). \tag{31}$$

If the union has full bargaining power, it chooses the combination of wages and employment on the labor demand curve that maximizes (31). Substituting out for the labor demand schedule $w_i = F_i$, the

[9]The minor distinction is that both these studies specify a function for the *unemployment* incidence, whereas for our purposes it is more convenient to use a function for the *employment* incidence.

[10]The reason for imposing this restriction is that when when $E_i = G_i(w_i - T_i + T_u)$ (i.e. whenever there is no involuntary unemployment), the notion of voluntary participation requires that rationing is efficient. This case corresponds to

$$e_i(E_i, \varphi) = \begin{cases} 1 & \text{if } \varphi \in [\underline{\varphi}_i, w_i - T_i + T_u] = [\underline{\varphi}_i, G_i^{-1}(E_i)] \\ 0 & \text{if } \varphi \in (w_i - T_i + T_u, \overline{\varphi}_i] = (G_i^{-1}(E_i), \overline{\varphi}_i], \end{cases}$$

which is not differentiable in the point $G_i^{-1}(E_i)$.

first-order condition, again assumed to be necessary and sufficient[11], is given by

$$\int_{\underline{\varphi}_i}^{\overline{\varphi}_i} e_i(E_i,\varphi)u'(w_i - T_i - \varphi)dG_i(\varphi)F_{ii} + \int_{\underline{\varphi}_i}^{\overline{\varphi}_i} e_i'(E_i,\varphi)(u(w_i - T_i - \varphi) - u(-T_u))dG_i(\varphi) = 0 \quad \Leftrightarrow$$

$$E_i\overline{v_i'} + E_i'(v_{i,r} - v_u) = 0. \qquad (32)$$

Again, $E_i' = 1/F_{ii}$ denotes the slope of the labor demand curve and we employ the property that, upon differentiating (30),

$$\int_{\underline{\varphi}_i}^{\overline{\varphi}_i} e_i'(E_i,\varphi)dG_i(\varphi) = 1. \qquad (33)$$

Since (33) integrates over nonnegative values to a measure of one, it specifies a distribution function. Using this insight, we define

$$v_{i,r} \equiv \int_{\underline{\varphi}_i}^{\overline{\varphi}_i} e_i'(E_i,\varphi)u(w_i - T_i - \varphi)dG_i(\varphi) \qquad (34)$$

as the *expected* utility of the individuals that are *rationed* whenever the rate of employment is marginally decreased. The intuition behind the first-order condition is almost identical to the one under the assumption of efficient rationing. The only difference lies in the fact that, with a general rationing scheme, the loss in employment is weighed by the expected utility loss of the rationed workers (i.e. $v_{i,r} - v_u$) instead of the utility loss of the marginal workers (i.e. $v_{i,m} - v_u$).

When combined with (5), the condition in (32) determines the equilibrium rate of employment in sector i when the union has full bargaining power. If, on the other hand, the union has no bargaining power at all, the labor market outcome again coincides with the competitive outcome, given by the combination of (5) and (11). Importantly, as this equilibrium does not feature any involuntary unemployment, the notion of voluntary participation ensures that rationing in this case is efficient.

Now, with a slight abuse of notation, we can write any equilibrium for a varying degree of the union's bargaining power $\rho_i \in [0,1]$ as a solution to the following equation

$$\rho_i E_i\overline{v_i'} + E_i'(v_{i,r} - v_u) = 0. \qquad (35)$$

When $\rho_i = 1$, (35) coincides with the (monopoly) union's first-order condition, as given by (32). However, the claim that for $\rho_i = 0$ the solution to the above expression coincides with the competitive outcome warrants explanation. To that end, first observe that if $\rho_i = 0$ (and labor demand is not perfectly elastic), (35) simplifies to $v_{i,r} = v_u$. In words, the expected utility of the rationed workers equals the well-being of the unemployed. Under the assumption that participation is voluntary and $e_i(E_i,\varphi)$ is nondecreasing, this can only happen when there is no involuntary unemployment, in which case rationing must

[11] At this point, a word of caution is required. Strictly speaking it is no longer guaranteed that the union *always* prefers an outcome with involuntary unemployment over the competitive outcome, as was the case with efficient rationing. This implies that the solution to the union's maximization problem need not be interior. For instance, if the individuals who lose their jobs first are the ones with the highest surplus from working (i.e. rationing is very inefficient), then it could be optimal for the union to refrain from demanding a wage that is above the market-clearing level. If this is the case, unions do not affect any of the labor market outcomes. Therefore, in order to keep things interesting, we assume in the remainder that the primitives of the model are such that the unions always have an incentive to increase the wage above the competitive level.

be efficient, which in turn implies that $v_{i,r} = v_{i,m}$. However, under efficient rationing, our definition of $v_{i,r}$ is strictly speaking no longer applicable as the corresponding rationing scheme is not differentiable (see footnote 10). Therefore, to avoid any inconsistencies, we set $v_{i,r}$ equal to $v_{i,m}$ whenever all unemployment is voluntary, which explains the aforementioned 'slight abuse of notation'. Then, if $\rho_i = 0$ the expression above reduces to $v_{i,m} = v_u$, which is indeed the equilibrium condition that applies when labor markets are competitive. Any intermediate equilibrium then corresponds to a specific choice for $\rho_i \in (0, 1)$ (in a similar fashion as displayed in Figure 1).

How does allowing for a more general rationing scheme affect the setting of the optimal income taxes and, more importantly, our result regarding the desirability of unions? The following Proposition answers these questions.

Proposition 4. *Consider the economy as described in Section 3 but now allow for a general rationing scheme. In the policy optimum, the tax schedule must satisfy*

$$N_i E_i(1 - b_i) + \sum_j N_j E_j(b_j - 1)\frac{\partial w_j}{\partial T_i} + \sum_j N_j \left[\frac{1}{\lambda}(v_{j,r} - v_u) + (T_j - T_u)\right]\frac{\partial E_j}{\partial T_i} = 0. \tag{36}$$

Furthermore, concerning the question how an increase in the bargaining power of the union representing type i workers affects social welfare, we again find that

$$\frac{\partial \mathcal{W}}{\partial \rho_i} > 0 \quad \Leftrightarrow \quad b_i > 1. \tag{37}$$

Proof. See Appendix D. □

Upon comparing (36) to (23), it is immediately clear that the condition for the optimal income taxes are hardly affected. The only notable difference is that, similarly to how the union's first-order condition is affected, the term $v_{i,m} - v_u$ is now replaced by $v_{i,r} - v_u$. This is a direct consequence from the generalization of the rationing scheme. The distortive impact of unions is no longer captured by the utility loss of the marginal workers, but instead by the expected utility loss of the rationed workers.

The second result stated in Proposition 4 has a remarkable implication: however the burden of involuntary unemployment (which results from the presence of unions) is allocated among workers with a different surplus from working, is completely irrelevant for the question whether or not unions can improve social welfare. This finding is in stark contrast to the result from Lee and Saez (2012) regarding the desirability of a minimum wage. As stated before, Gerritsen and Jacobs (2014) show the way in which unemployment is allocated among workers with different participation costs matters crucially for the question whether or not the introduction of a binding minimum wage for low-skilled workers is welfare-enhancing. To understand why this is not the case with unions, note that, while both unions and minimum wages may lead to involuntary unemployment, these institutions differ crucially along two other dimensions. Firstly, unlike minimum wages, unions have the potential to internalize any inefficiencies related to the rationing scheme. This can most clearly be seen from (32). If rationing is very inefficient, this will refrain the union from demanding a wage that is far above the market-clearing level. Obviously, such a mechanism is absent when a minimum wage is introduced. Secondly and most importantly, unlike with minimum wages, under unionized wage-setting changes in the tax system will still have an impact on labor market outcomes. To see why this is true, consider Figure 1. If there is a wage floor (i.e. a binding minimum wage), a marginal change in either the income tax or the unemployment benefit will change the positioning of the labor supply curve, but this policy intervention will neither affect the wage, nor the rate of employment. With unions, however, the story is radically different. When there is a change in either the income tax or the unemployment benefit, unions respond optimally by adapting their wage claim. This implies that in the presence of unions, as opposed to the case with minimum wages, the government still has scope to use its tax instruments to affect labor market outcomes. And it is exactly for this reason that the result regarding the desirability of unions is not sensitive to the specifics of the rationing scheme.

To illustrate the above argument in a bit more detail, suppose that there is an increase in the bargaining power of a union representing low-skilled workers. If the government does not respond by changing its tax and benefit system, the increase in the union's bargaining power puts upward pressure on the wage and downward pressure on the rate of employment. In this regard, an increase in the degree of unionization has a very similar impact as increasing the minimum wage. However, unlike with a binding minimum wage, under unionized wage-setting the government can use its tax instruments to fully offset the impact of an increased degree of unionization on any of the labor market outcomes (recall our discussion from Section 5). And, importantly, this reasoning holds true *irrespective of the specifics of the rationing scheme*. Consequently, the question whether or not unions are a desirable institution for redistribution is not affected by any considerations regarding the question which workers bear the burden of unemployment.

Finally, it is insightful to think a bit more carefully about how minimum wages and unions affect the government's scope to use its tax and benefit system in order to implement its preferred allocation. Lee and Saez (2012) show that the allocation that can be achieved by combining the tax and benefit system with minimum wages cannot be replicated by means of the tax and benefit system alone. Introducing a minimum wage can therefore be welfare-enhancing, provided that the rationing scheme is not too inefficient (Gerritsen and Jacobs, 2014). As explained in Section 2, this is because minimum wages, unlike the tax and benefit system, enable the government to create involuntary unemployment. And as long as the costs of involuntary unemployment remain sufficiently low (which is the case only if rationing is not too inefficient), increasing the minimum wage for low-skilled worker improves social welfare. Now, since our main result regarding the desirability of unions continues to hold for a general, and potentially highly inefficient rationing scheme, it is directly implied that the allocation that can be implemented through the combination of taxes and unions cannot be replicated by either (i) the tax and benefit system alone, or (ii) the combination of the tax and benefit system and a minimum wage. The reason is that, as mentioned before, in the presence of unions the government can still use its tax and benefit system to influence the unions' wage claims and thereby affect labor market outcomes. This enables the government to offset any impact from stronger unions on employment, a mechanism which the government lacks whenever there is binding minimum wage. The above discussion thus illustrates that unions, when compared to minimum wages, are potentially a much more useful institution to enhance redistribution towards low-skilled workers.

7 Extensions

In this section, we study two extensions of the model we analyzed thus far. Firstly, we investigate the consequences of a binding restriction on profit taxation. Any policy optimum that has been characterized up to this point features the property that the welfare weight of firm-owners equals one. In other words, in the policy optimum the government is always indifferent between receiving an additional unit of resources R and increasing the firm-owners' consumption by one unit. This result directly follows from our assumption that the government could 'freely' (i.e. in a non-distortive way) tax profits up to the point it desires. This may not be particularly realistic, as in reality the government may face considerable challenges when aiming to tax profits, for instance because of the tax-avoidance strategies or because of international tax competition (Fuest and Huber, 1997).

Secondly, we will also analyze the case where the government has a *Rawlsian* objective. As stated before, with Rawlsian social preferences, the government only attaches a positive weight to the well-being of workers who are least well-off. This, in turn, may affect the *qualitative* nature of our results. Indeed, something similar is observed in Saez (2002), who shows that the question whether it can *ever* be optimal to subsidize participation on a net basis depends crucially on the redistributive preferences of the government. We will examine whether something similar is observed in a model where wages are endogenously determined in unionized labor markets.

Without further notice, we assume in the remainder that labor markets are independent and labor demand is not perfectly elastic. Hence, production can be thought to be described by (14). The reason for doing so is that any interdependencies between labor markets are not of first-order importance for either of the extensions presented next. Furthermore, recall from our previous discussion that with perfectly elastic labor demand, there will be no role for unions whatsoever, so that again this case is not of primary interest. Finally, we assume that rationing is as described in the previous section (i.e. we allow for a general rationing scheme).

7.1 Restricted Profit Taxation

Throughout we have assumed that the government could levy a fully non-distortive profit tax. Since this tax did not affect any of the decisions made by the agents in the private sector, the government would always tax profits up to the point where the welfare weight of firm-owners equals one.

To investigate what would happen if this is no longer possible, let us assume that the government faces a restriction on profit taxation of the form $T_f \leq T_f^*$. To keep things interesting, let us furthermore assume that this constraint is binding in the policy optimum. It is shown in Appendix E that in this case, the welfare weight of firm-owners will fall short of one, i.e. $b_f < 1$. In words, the government values an additional unit of resources R more than it values an increase in the firm-owners' consumption by this same unit whenever profit taxation is restricted. The consequences of this additional restriction for the optimal tax and benefit system and for the question whether unions are a desirable institution for redistribution are summarized in the next Proposition.

Proposition 5. *Suppose that the government faces a binding restriction on the extent to which profits can be taxed (of the form $T_f \leq T_f^*$) and consider the case where labor markets are independent and labor* *demand is not perfectly elastic. Then, in the policy optimum,*

(i) the optimal tax schedule satisfies

$$T_i - T_u = -\frac{1}{\lambda}(v_{i,r} - v_u) + \frac{w_i}{\varepsilon_{wi}}(b_f - b_i) - \frac{T_i}{\varepsilon_{Ti}}(1 - b_i), \tag{38}$$

(ii) increasing the bargaining power of workers is accompanied by an increase in social welfare if and only if the workers' welfare weight exceeds one (i.e. $\partial \mathcal{W}/\partial \rho_i > 0 \Leftrightarrow b_i > 1$).

Proof. See Appendix E. □

The second point from Proposition 5 establishes that our main result regarding the desirability of unions is not affected by a restriction on profit taxation. This should come as no surprise. As argued previously, stronger unions allow for more redistribution towards the (employed) workers who are organized in a union. Therefore, increasing the bargaining power of a union is socially desirable as long as the government values a unit of redistribution towards these workers more than it values its own resources. A binding restriction on profit taxation simply does not interfere with this result.

Secondly, with respect to our expression for the optimal participation tax, we see one notable difference when comparing (38) to (24). In particular, the term involving the elasticity of labor demand is no longer weighed by $1 - b_i$, but rather by $b_f - b_i$. This result is intuitive. As was argued in Section 4, this term captures the welfare impact of a change in the wage, which constitutes a change in the transfer from firm-owners to workers. When there is no restriction on profit taxation, the former's welfare weight equals one and hence the welfare effect was weighed by $1 - b_i$. In the presence of such a restriction, however, the associated welfare effect is proportional to $b_f - b_i < 1 - b_i$. Not surprisingly, the above result states that the income tax should *ceteris paribus* be higher the more binding is the restriction on profit taxation (i.e. the lower is T_f^* and consequently b_f). As argued before, a higher income tax motivates the union to increase its wage claim, which brings about additional redistribution of income from firm-owners to workers.

7.2 Rawlsian Social Preferences

As argued before, Diamond (1980) and Saez (2002) show that when wages are exogenous (or labor markets are competitive, as in Christiansen, 2015), subsidizing participation on a net basis (i.e. setting $T_i - T_u < 0$) can only be socially desirable for low-skilled workers, i.e. workers whose welfare weight exceeds one. As shown in Section 4, this result does not generalize to a setting where the degree of unionization is nonzero. In particular, with strong unions, it may be optimal to subsidize participation even for high-skilled workers. This results from the fact that, in line with what was explained in Section 4, when wage-setting is unionized the tax and benefit system also serves to alleviate the distortions induced by unions.

In deriving the above results we have thus far assumed that the government has utilitarian social preferences. Saez (2002), on the other hand, allows for more general social preferences and argues that, whenever the government has a Rawlsian objective, subsidizing participation on a net basis can *never* be optimal. This can readily be seen from (27), where we restated the result from Saez (2002). If the government only cares about the individuals that are worst off (typically the unemployed), then the welfare weight of all types of employed workers is equal to zero (i.e. $b_i = 0$ for all i) and participation is never subsidized on a net basis. We will now investigate whether the finding that a Rawlsian government will always refrain from subsidizing participation generalizes to a setting where labor markets are unionized, in which case taxes also serve to alleviate distortions induced by unions.

To operationalize ideas, we first have to reconsider the government's optimization problem. In particular, with Rawlsian preferences, the objective (7) is now replaced by

$$\mathcal{W} = \min\{\{v_i(\varphi)\}_{i\in\mathcal{I}}, v_u, v_f\}. \tag{39}$$

In words, social welfare is determined solely by the well-being of the individuals in the economy that are worst off. The government's problem is to maximize (39) subject to the budget constraint (6) the firm's optimality conditions (5) and the labor market equilibrium conditions. For a general rationing scheme, the latter are given by (35). Since we always assume that participation is voluntary, it must be that $v_i(\varphi) \geq v_u$. Consequently, the expression in (39) reduces to either v_u or v_f. Now, one way to write the government's problem is as follows (again, using the firm's optimality conditions to substitute out for the wages):

$$\begin{aligned}
\max_{T_1,\dots,T_I,T_u,T_f,E_1,\dots,E_I} \mathcal{L} &= u(-T_u) + \kappa\left(F - \sum_i F_i N_i E_i - T_f + T_u\right) \\
&+ \lambda(R + \sum_i N_i(E_i T_i + (1-E_i)T_u) + T_f) + \sum_i \mu_i \left(\rho_i \int_{\underline{\varphi}_i}^{\overline{\varphi}_i} e_i(E_i,\varphi) u_i'(F_i - T_i - \varphi) dG_i(\varphi) F_{ii} \right. \\
&\left. + \int_{\underline{\varphi}_i}^{\overline{\varphi}_i} e_i'(E_i,\varphi)(u_i(F_i - T_i - \varphi) - u(-T_u)) dG_i(\varphi)\right).
\end{aligned} \tag{40}$$

In words, the government maximizes the well-being of the unemployed, subject to the constraint that the consumption of the firm-owners is at least as high as that of the unemployed (which is equivalent to stating that $v_f \geq v_u$), in addition to the budget restriction and the labor market equilibrium constraints (which ensure that $v_i(\varphi) \geq v_u$ is always satisfied). The first-order conditions associated to the problem described by (40) can be found in Appendix F. They can be directly manipulated to obtain the following results.

Proposition 6. *Suppose that the government has Rawlsian social preferences and consider the case where labor markets are independent and labor demand is not perfectly elastic. Then, in the policy optimum,*

(i) firm-owners and unemployed workers are equally well off (i.e. $v_f = v_u$),

(ii) participation is never subsidized on a net basis (i.e. $T_i > T_u$ for all i),

(iii) increasing the bargaining power of workers is always accompanied by a decrease in social welfare (i.e. $\partial\mathcal{W}/\partial\rho_i < 0$ for all i).

Proof. See Appendix F. □

The first of these results states that a Rawlsian government will redistribute from firm-owners to unemployed workers up to the point where they are equally well-off. As before, this follows from the fact that the government can tax firm-owners in a lump-sum way. The intuition behind the second result is also fairly straightforward. As the government does not value any redistribution towards the working, it is never optimal to subsidize participation on a net basis. Importantly, this is true despite the fact that unions cause distortions, which we argued previously (with utilitarian preferences) could be a reason to subsidize participation even for workers whose welfare weight is below one. The reason why this mechanism is absent with a Rawlsian government is that a decrease in involuntary unemployment has no direct welfare effect from the government's perspective. While an involuntarily unemployed worker strictly benefits from finding a job, the direct increase in his or her well-being is simply not valued by a Rawlsian government. Hence, there is no direct motive for the government to use its tax and benefit system to lower involuntary unemployment (i.e. to reduce the distortions induced by unions).

Finally, the third result from Proposition 6 implies that a Rawlsian government always prefers competitive over unionized labor markets. This may seem somewhat paradoxical at first sight, as one would arguably not expect a more 'left-wing' government to advocate lowering the bargaining power of *all* workers relative to that of firm-owners. The intuition, however, is quite straightforward. A Rawlsian government wants to redistribute as much as possible to the unemployed. Obviously, more can be redistributed towards unemployed workers the fewer unemployed workers there are. And since stronger unions lead to higher rates of unemployment (at least, for a given level of taxation), they naturally worsen the scope to redistribute income towards the unemployed. Hence, whereas a utilitarian government prefers to have unions at the lower part of the income distribution (provided there is a group of employed workers whose welfare weight exceeds one), a Rawlsian government does not value any additional redistribution towards low-skilled workers and simply prefers wages to be competitively determined rather than in a bargaining fashion between unions and firm-owners.

8 Concluding Remarks

The aim of this paper has been to answer two questions concerning optimal income redistribution with unionized labor markets. With respect to the question *"How should the government optimize income redistribution when labor markets are unionized and labor supply responds along the extensive margin?"*, our most important finding is that the optimal tax and benefit system is not only used to redistribute income, but also serves to alleviate the distortions induced by unions. In particular, we show that income taxes should be lower the larger are the welfare gains associated to lowering involuntary unemployment. Intuitively, low income taxes motivate the unions to moderate the (pretax) wage claims, which results in less involuntary unemployment. Whenever the welfare gains associated to lowering involuntary unemployment are high, the government should exploit this channel and set low income taxes. And it is exactly because of this channel that it may be optimal to subsidize participation on a net basis (i.e. setting an income subsidy that exceeds the unemployment benefit) even for workers whose welfare weight exceeds one, something that can never be optimal when labor markets are competitive (see, e.g. Diamond, 1980, Saez, 2002, Christiansen, 2015).

Concerning the question *"Are unions a useful institution for redistribution?"*, our most important result is that it is socially desirable to increase the bargaining power of the unions representing low-income workers, whereas the opposite holds true for high-income workers. Intuitively, when wage-setting for low-skilled workers is unionized, the government can use its tax and benefit system more efficiently to achieve a certain degree of redistribution than what can be achieved with competitive labor markets. To illustrate this, consider a situation with perfectly competitive labor markets where the government has optimally set its tax instruments. Because participation costs are unobservable, the corresponding allocation is not first-best. Now, suppose that low-skilled workers find a way to organize themselves in a union. Naturally, the introduction of a union puts upward pressure on the pretax wage for low-skilled workers, which harms employment. The government, however, can perfectly offset the impact on the pretax wage (and hence employment) by lowering the income tax levied on low-skilled workers (as a decrease in the income tax would motivate the recently established union to moderate the wage claim). The latter, in turn, can be financed by increasing taxes elsewhere in the economy. Unions thus allow the government to bring about additional redistribution towards the workers who are organized in a union, on top of what can be achieved when labor markets are competitive. And it is exactly for this reason that it is socially desirable to let low-skilled workers organize themselves in a union, whereas the wages for the more productive workers should preferably be determined competitively. Furthermore, it should be emphasized that the above argumentation holds true, irrespective of the question which workers bear the burden of involuntary unemployment. In that respect, our result regarding the desirability of unions differs fundamentally from the Lee and Saez (2012) result regarding the desirability of a minimum wage. As shown in Gerritsen and Jacobs (2014), the finding that a minimum wage may improve social welfare hinges crucially on the assumption that rationing is efficient, which means that the workers who become unemployed are the ones with the highest participation costs. The reason why any specifics regarding the rationing scheme are irrelevant for the question whether unions are a useful institution for redistribution is that, unlike with minimum wages, in the presence of unions the government can still use its tax and benefit system to affect labor market outcomes.

In deriving the above results, we have made several simplifying assumptions. We will discuss the two we believe are most crucial. Firstly, we have assumed throughout that the government is the Stackelberg leader relative to all agents in the private sector, *including the unions*. This assumption has been crucial in deriving the result that unions are a useful institution for redistributive purposes, provided that they represent low-skilled workers. However, this assumption has not gone uncontested in the literature. In particular, Boeters and Schneider (1999) and Aronsson and Wikström (2011), among others, also consider the case where the union is the Stackelberg leader and show that the structure of the game has important implications for (optimal) tax policy. Nevertheless, what should be noted is that in both these studies there is only one type of labor and consequently only one union, which is furthermore assumed to have full bargaining power relative to the firms. On the contrary, our model features many unions, all of whom vary in terms of their bargaining power relative to firm-owners. In such a setting, we believe it seems most natural as well as most realistic to assume that the government is the Stackelberg leader relative to the actors in the private sector (see Palokangas, 1987, for a more detailed exposition of this argument).

Secondly, we have abstracted from any intensive margin considerations: workers could not choose their occupation, neither could they (or the union) affect the number of hours worked. And while this is obviously a crude abstraction from reality, there is another, less obvious reason why this modeling choice may not be innocuous in the present context. By following the convention from the optimal taxation literature with purely extensive labor supply responses (see, e.g., Diamond, 1980, Saez, 2002, Choné and Laroque, 2011, Christiansen, 2015) to let the government directly choose a level of taxation at each point in the wage distribution (which, in our model, boils down to picking the average tax for each type of worker rate, while setting the marginal tax rate equal to zero), there is only an *income effect* (and no *substitution effect*) associated to the impact of taxation on labor market outcomes. To illustrate what we mean by this, consider the following. In an environment where unions face a trade-off between wages and employment, an increase in the average tax rate (keeping the marginal tax rate constant) motivates unions to increase the wage claim in order to compensate the employed workers, which constitutes an income effect. If, on the other hand, the government increases the marginal tax rate (leaving the average tax rate unaffected), the unions will moderate their wage claim, as through a higher marginal tax rate the unions' trade-off is shifted in favor of employment. The latter describes a substitution effect. In the model considered in this study, only the first of those effects is present. Consequently, the model gives a too simplistic view of how taxes influence the wage claims of the unions and consequently, labor market outcomes. Therefore, extending the model to incorporate an intensive margin (combined with a tax system that allows for nonzero marginal tax rates) and analyzing the consequences for optimal redistributive taxation appears to be a highly interesting avenue for future research.

As a final remark, we strongly advocate further empirical research on the questions we considered in this paper. For instance, it would be of great interest to investigate how the presence of unions affects the (shape of the) optimal tax schedule, as graphically depicted for the U.S. in Jacquet et al. (2013) and for the Netherlands in Zoutman et al. (2013) (both of which consider a setting with intensive as well as extensive labor supply responses and exogenous wages). Furthermore, quantifying the potential welfare gains that can be brought about from increasing the bargaining power of the unions representing low-income workers could also be of considerable academic interest, as well as policy relevance.

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Appendices

A Proof Proposition 1

The Lagrangean associated to the government's problem reads

$$\begin{aligned}\mathcal{L} = &\sum_i N_i \left(\int_{\underline{\varphi}_i}^{G_i^{-1}(E_i)} u(F_i - T_i - \varphi) dG_i(\varphi) + \int_{G_i^{-1}(E_i)}^{\overline{\varphi}_i} u(-T_u) dG_i(\varphi) \right) \\ &+ u(F - \sum_i F_i N_i E_i - T_f) + \lambda(R + \sum_i N_i(E_i T_i + (1 - E_i)T_u) + T_f) \\ &+ \sum_i \mu_i \left(\rho_i \int_{\underline{\varphi}_i}^{G_i^{-1}(E_i)} u'(F_i - T_i - \varphi) dG_i(\varphi) F_{ii} + u(F_i - T_i - G_i^{-1}(E_i)) - u(-T_u) \right). \end{aligned} \quad (41)$$

The first-order conditions are given by

$$
\begin{aligned}
T_i: &\quad N_i E_i(\lambda - \overline{v_i'}) - \mu_i \left(\rho_i E_i \overline{v_i''} F_{ii} + v_{i,m}'\right) = 0 && (42)\\
T_u: &\quad \sum_i N_i(1-E_i)(\lambda - v_u') + \sum_i \mu_i v_u' = 0 && (43)\\
T_f: &\quad \lambda - v_f' = 0 && (44)\\
E_i: &\quad N_i(v_{i,m} - v_u) + N_i \sum_j E_j(\overline{v_j'} - v_f')F_{ji} + \lambda N_i(T_i - T_u) \\
&\quad + N_i \sum_j \mu_j \left(\rho_j E_j \overline{v_j''} F_{jj} F_{ji} + \rho_j E_j \overline{v_j'} F_{jji} + v_{j,m}' F_{ji}\right)/N_j + \mu_i(\rho_i v_{i,m}' F_{ii} - v_{i,m}'/G_i') = 0 && (45)\\
\lambda: &\quad R + \sum_i N_i(E_i T_i + (1-E_i)T_u) + T_f = 0 && (46)\\
\mu_i: &\quad \rho_i E_i \overline{v_i'} F_{ii} + v_{i,m} - v_u = 0. && (47)
\end{aligned}
$$

Point (i) from Proposition 1 directly follows from (44) and our definition of b_f. To prove point (ii), use (42) to substitute out for all μ_i's in (43). Then, collect the terms with λ on the right-hand side, divide the resulting expression by λ and impose the definitions for b_i and b_u. After rearranging, one obtains the result stated in the Proposition, with weights given by

$$
\omega_i \equiv \frac{N_i E_i v_u'}{\rho_i E_i \overline{v_i''} F_{ii} + v_{i,m}'} \left(\sum_j N_j \left[\frac{E_j v_u'}{\rho_j E_j \overline{v_j''} F_{jj} + v_{j,m}'} + (1 - E_j) \right] \right)^{-1} \tag{48}
$$

$$
\omega_u \equiv \sum_i N_i(1 - E_i) \left(\sum_j N_j \left[\frac{E_j v_u'}{\rho_j E_j \overline{v_j''} F_{jj} + v_{j,m}'} + (1 - E_j) \right] \right)^{-1}. \tag{49}
$$

When $\rho_i = 0$ for all i, (47) implies that $v_{i,m} = v_u$ and hence $v_{i,m}' = v_u'$. Substituting this in (48)-(49) reduces the weights to

$$
\omega_i = \frac{N_i E_i}{\sum_j N_j} \tag{50}
$$

$$
\omega_u = \frac{\sum_i N_i(1 - E_i)}{\sum_j N_j}, \tag{51}
$$

which are just the shares in the total labor force, as stated in the text.

B Proof Proposition 2 and Corollary 1

Proof Proposition 2

To prove the first part, note that we can use the labor market equilibrium conditions and the firm's optimality conditions to write $E_i = E_i(T_1, .., T_I, T_u)$ and $w_i = w_i(T_1, .., T_I, T_u)$. The Lagrangean then reads:

$$
\begin{aligned}
\mathcal{L} = & \sum_i N_i \left(\int_{\underline{\varphi}_i}^{G_i^{-1}(E_i(T_1,..,T_I,T_u))} u(w_i(T_1,..,T_I,T_u) - T_i - \varphi) dG_i(\varphi) \right. \\
& \left. + \int_{G_i^{-1}(E_i(T_1,..,T_I,T_u))}^{\overline{\varphi}_i} u(-T_u) dG_i(\varphi) \right) \\
& + u(F(K, N_1 E_1(T_1,..,T_I,T_u),.., N_I E_I(T_1,..,T_I,T_u)) \\
& - \sum_i w_i(T_1,..,T_I,T_u) N_i E_i(T_1,..,T_I,T_u) - T_f) \\
& + \lambda(R + \sum_i N_i(E_i(T_1,..,T_I,T_u)T_i + (1 - E_i(T_1,..,T_I,T_u))T_u) + T_f).
\end{aligned} \tag{52}
$$

Differentiating with respect to T_i and T_f yields

$$
T_i: \quad N_i E_i(\lambda - \overline{v_i'}) + \sum_j N_j E_j(\overline{v_j'} - v_f') \frac{\partial w_j}{\partial T_i} + \sum_j N_j \left[(v_{j,m} - v_u) + \lambda(T_j - T_u) \right] \frac{\partial E_j}{\partial T_i} = 0 \tag{53}
$$

$$
T_f: \quad \lambda - v_f' = 0. \tag{54}
$$

Now, to arrive at the first result stated in the Proposition, use (54) to substitute out for v_f' in (53) and divide the resulting expression by λ.

The second part is proven as follows. Consider (45) and impose that $F_{ij} = 0$ for all $i \neq j$. We then obtain:

$$
\begin{aligned}
& N_i(v_{i,m} - v_u) + N_i E_i(\overline{v_i'} - v_f') F_{ii} + \lambda N_i(T_i - T_u) \\
& + \mu_i \left(\rho_i E_i \overline{v_i''} F_{ii} F_{ii} + \rho_i E_i \overline{v_i'} F_{iii} + v_{i,m}' F_{ii} + \rho_i v_{i,m}' F_{ii} - v_{i,m}'/G_i' \right) = 0.
\end{aligned} \tag{55}
$$

Again, use the property that $v_f' = \lambda$ to substitute out for v_f', divide the resulting expression by λ and substitute out for μ_i using (42). After rearranging,

$$
\begin{aligned}
T_i - T_u = & - \frac{1}{\lambda}(v_{i,m} - v_u) + E_i F_{ii}(1 - b_i) \\
& - \left(E_i \frac{\rho_i E_i \overline{v_i''} F_{ii} F_{ii} + \rho_i E_i \overline{v_i'} F_{iii} + v_{i,m}' F_{ii} + \rho_i v_{i,m}' F_{ii} - v_{i,m}'/G_i'}{\rho_i E_i \overline{v_i''} F_{ii} + v_{i,m}'} \right) (1 - b_i).
\end{aligned} \tag{56}
$$

The expression for the elasticity of labor demand can be found by implicitly differentiating the firm's first-order condition:

$$
\varepsilon_{wi} = \frac{\partial E_i}{\partial w_i} \frac{w_i}{E_i} = \frac{1}{F_{ii}} \frac{w_i}{E_i} < 0, \tag{57}
$$

provided that labor demand is not perfectly elastic (in which case $F_{ii} = 0$). In addition, the elasticity of the equilibrium employment rate with respect to the income tax can be found by implicitly differentiating the labor market equilibrium condition:

$$
\varepsilon_{Ti} = \frac{\partial E_i}{\partial T_i} \frac{T_i}{E_i} = \frac{\rho_i E_i \overline{v_i''} F_{ii} + v_{i,m}'}{\rho_i E_i \overline{v_i''} F_{ii} F_{ii} + \rho_i E_i \overline{v_i'} F_{iii} + v_{i,m}' F_{ii} + \rho_i v_{i,m}' F_{ii} - v_{i,m}'/G_i'} \frac{T_i}{E_i} < 0, \tag{58}
$$

where the sign follows from the strict concavity of the union's objective. Combining (56)-(58), we obtain

$$
T_i - T_u = -\frac{1}{\lambda}(v_{i,m} - v_u) + \left(\frac{w_i}{\varepsilon_{wi}} - \frac{T_i}{\varepsilon_{Ti}} \right) (1 - b_i). \tag{59}
$$

What remains to be shown is that the term multiplied by $1-b_i$ in (59) is strictly positive. To that end, consider

$$\frac{w_i}{\varepsilon_{wi}} - \frac{T_i}{\varepsilon_{Ti}} = \frac{-E_i\left(\rho_i E_i \overline{v_i'} F_{iii} + v_{i,m}' F_{ii} - v_{i,m}'/G_i'\right)}{\rho_i E_i \overline{v_i''} F_{ii} + v_{i,m}'} > 0 \tag{60}$$

after simplifying and again employing the fact that the union's objective is assumed to be strictly concave in E_i (which ensures that the first-order condition is both necessary and sufficient).

Proof Corollary 1

Consider (45). As before, substitute out for v_f' using (44), divide the resulting expression by λ and use (42) to substitute out for all the μ_j's. After rearranging,

$$\begin{aligned} T_i - T_u = &- \frac{1}{\lambda}(v_{i,m} - v_u) \\ &+ \left(E_i F_{ii} - E_i \frac{\rho_i E_i \overline{v_i''} F_{ii} F_{ii} + \rho_i E_i \overline{v_i'} F_{iii} + v_{i,m}' F_{ii} + \rho_i v_{i,m}' F_{ii} - v_{i,m}'/G_i'}{\rho_i E_i \overline{v_i''} F_{ii} + v_{i,m}'}\right)(1-b_i) \\ &+ \sum_{j \neq i}\left(E_j F_{ji} - E_j \frac{\rho_j E_j \overline{v_j''} F_{jj} F_{ji} + \rho_j E_j \overline{v_j'} F_{jji} + v_{j,m}' F_{ji}}{\rho_j E_j \overline{v_j''} F_{ji} + v_{j,m}'}\right)(1-b_j), \end{aligned} \tag{61}$$

which generalizes (56). In order to prove (i), first note that the assumptions of constant returns to scale and cooperative factors of production imply that $F_{ij} = 0$ for all $i \neq j$ whenever $F_{ii} = 0$ for all i. To see why this is true, use the linear homogeneity of the production function to derive that

$$-F_{ii}(K, L_1, .., L_I)L_i = \sum_{j \neq i} F_{ji}(K, L_1, .., L_I)L_j + F_{Ki}(K, L_1, .., L_I)K. \tag{62}$$

If $F_{ii} = 0$ and factors are cooperative in production (i.e. $F_{Ki}, F_{ji} \geq 0$), from (62) it must be that $F_{ji} = F_{Ki} = 0$ as well. Furthermore, from (47) it directly follows that $v_{i,m} = v_u$ whenever $F_{ii} = 0$. Substituting all this in (61), the expression reduces to

$$T_i - T_u = \frac{E_i}{G_i'}(1-b_i). \tag{63}$$

Finally, from $v_{i,m} = v_u$ and the definition $v_{i,m}$ it follows that $E_i = G_i$. After plugging this in (63), the resulting expression coincides with the one stated in Saez (2002).

The proof of point (ii) is almost identical. Observe that also when $\rho_i = 0$, from (47) it must be that $v_{i,m} = v_u$. Plugging this in (61) (together with $\rho_i = 0$ for all i), we again obtain the result stated in (63).

C Proof Proposition 3 and Corollary 2

Proof Proposition 3

To determine how a change in ρ_i affects social welfare, differentiate (41) with respect to ρ_i and use the Envelope theorem:

$$\frac{\partial \mathcal{W}}{\partial \rho_i} = \frac{\partial \mathcal{L}}{\partial \rho_i} = \mu_i N_i E_i \overline{v_i'} F_{ii}. \tag{64}$$

Since $N_i E_i \overline{v_i'} F_{ii} < 0$ (provided that labor demand is not perfectly elastic), the expression in (64) is positive if and only if $\mu_i < 0$. Next, consider expression (42). After some rearranging,

$$1 - b_i = \frac{\mu_i}{\lambda N_i E_i} \left(\rho_i E_i \overline{v_i''} F_{ii} + v'_{i,m} \right). \tag{65}$$

Since $\lambda N_i E_i > 0$ and $\rho_i E_i \overline{v_i''} F_{ii} + v'_{i,m} > 0$, it must be that

$$\mu_i < 0 \quad \Leftrightarrow \quad b_i > 1. \tag{66}$$

Hence, an increase in ρ_i leads to an increase in social welfare if and only if $b_i > 1$. This completes the proof.

Proof Corollary 2

Suppose that the government could also optimally set the bargaining power of each union ρ_i in addition to the tax instruments. Denote by $\underline{\kappa}_i \geq 0$ the Kuhn-Tucker multiplier on the restriction that $\rho_i \geq 0$ and by $\overline{\kappa}_i \geq 0$ the multiplier on the restriction that $1 - \rho_i \geq 0$. Then, the first-order condition with respect to ρ_i is given by

$$\mu_i N_i E_i \overline{v_i'} F_{ii} + \underline{\kappa}_i - \overline{\kappa}_i = 0, \tag{67}$$

which should be considered alongside the first-order conditions given in Appendix A. In an interior optimum (i.e. where the optimal $\rho_i^* \in (0,1)$), the Kuhn-Tucker conditions require that $\underline{\kappa}_i = \overline{\kappa}_i = 0$, which by (67) requires that $\mu_i = 0$ (again provided that labor demand is not perfectly elastic). Then, from (42), it follows that $b_i = 1$ in any interior optimum.

If the solution is at the boundary, then by the Kuhn-Tucker conditions it must be that either $\overline{\kappa}_i = 0$ and $\underline{\kappa}_i > 0$ or $\underline{\kappa}_i = 0$ and $\overline{\kappa}_i > 0$. Whenever labor demand is not perfectly elastic, (67) implies that $\mu_i > 0$ in the first case (which by (42) requires that $b_i < 1$) and $\mu_i < 0$ in the second case (which requires by (42) that $b_i > 1$). This establishes the result.

D Proof Proposition 4

To prove the first part, note that again we can use the labor market equilibrium conditions and the firm's optimality conditions to write $E_i = E_i(T_1, .., T_I, T_u)$ and $w_i = w_i(T_1, .., T_I, T_u)$. With a general rationing scheme, the Lagrangean then reads:

$$\begin{aligned}
\mathcal{L} = \sum_i N_i \int_{\underline{\varphi}_i}^{\overline{\varphi}_i} & (e_i(E_i(T_1, .., T_I, T_u), \varphi) u(w_i(T_1, .., T_I, T_u) - T_i - \varphi) \\
& + (1 - e_i(E_i(T_1, .., T_I, T_u), \varphi)) u(-T_u)) \, dG_i(\varphi) \\
& + u(F(K, N_1 E_1(T_1, .., T_I, T_u), .., N_I E_I(T_1, .., T_I, T_u)) \\
& - \sum_i w_i(T_1, .., T_I, T_u) N_i E_i(T_1, .., T_I, T_u) - T_f) \\
& + \lambda(R + \sum_i N_i (E_i(T_1, .., T_I, T_u) T_i + (1 - E_i(T_1, .., T_I, T_u)) T_u) + T_f).
\end{aligned} \tag{68}$$

The first-order conditions with respect to T_i and T_f are given by

$$T_i: \quad N_iE_i(\lambda - \overline{v_i'}) + \sum_j N_jE_j(\overline{v_j'} - v_f')\frac{\partial w_j}{\partial T_i} + \sum_j N_j\left[(v_{j,r} - v_u) + \lambda(T_j - T_u)\right]\frac{\partial E_j}{\partial T_i} = 0 \tag{69}$$

$$T_f: \quad \lambda - v_f' = 0, \tag{70}$$

using the definition of $v_{i,r}$ from the main text. Combining (69)-(70) directly yields the result stated in the Proposition.

In order to prove the second part, we first modify the Lagrangean that explicitly takes into account the labor market equilibrium constraints to accommodate for a general rationing scheme:

$$\begin{aligned}\mathcal{L} = &\sum_i N_i\left(\int_{\underline{\varphi}_i}^{\overline{\varphi}_i}(e_i(E_i,\varphi)u(F_i - T_i - \varphi) + (1 - e_i(E_i,\varphi))u(-T_u))\,dG_i(\varphi)\right)\\ &+ u(F - \sum_i F_iN_iE_i - T_f) + \lambda(R + \sum_i N_i(E_iT_i + (1 - E_i)T_u) + T_f)\\ &+ \sum_i \mu_iN_i\left(\rho_i\int_{\underline{\varphi}_i}^{\overline{\varphi}_i} e_i(E_i,\varphi)u_i'(F_i - T_i - \varphi)dG_i(\varphi)F_{ii}\right.\\ &\left.+ \int_{\underline{\varphi}_i}^{\overline{\varphi}_i} e_i'(E_i,\varphi)(u_i(F_i - T_i - \varphi) - u(-T_u))dG_i(\varphi)\right),\end{aligned} \tag{71}$$

which is the counterpart of (41). The proof is now identical to that of Proposition 3 (see Appendix C), with the only distinction that the Lagrangean given by (41) is replaced by (71).

E Proof Proposition 5

With restricted profit taxation, the Lagrangean associated to the government's problem reads

$$\begin{aligned}\mathcal{L} = &\sum_i N_i\left(\int_{\underline{\varphi}_i}^{\overline{\varphi}_i}(e_i(E_i,\varphi)u(F_i - T_i - \varphi) + (1 - e_i(E_i,\varphi))u(-T_u))\,dG_i(\varphi)\right)\\ &+ u(F - \sum_i F_iN_iE_i - T_f) + \lambda(R + \sum_i N_i(E_iT_i + (1 - E_i)T_u) + T_f)\\ &+ \sum_i \mu_iN_i\left(\rho_i\int_{\underline{\varphi}_i}^{\overline{\varphi}_i} e_i(E_i,\varphi)u_i'(F_i - T_i - \varphi)dG_i(\varphi)F_{ii}\right.\\ &\left.+ \int_{\underline{\varphi}_i}^{\overline{\varphi}_i} e_i'(E_i,\varphi)(u_i(F_i - T_i - \varphi) - u(-T_u))dG_i(\varphi)\right) + \nu(T_f^* - T_f),\end{aligned} \tag{72}$$

where $\nu > 0$ denotes the multiplier on the profit taxation constraint (assumed to be binding). Under the assumption that labor markets are independent, the first-order conditions are given by

$$T_i: \quad N_i E_i(\lambda - \overline{v_i'}) - \mu_i \left(\rho_i E_i \overline{v_i''} F_{ii} + v_{i,m}'\right) = 0 \tag{73}$$

$$T_u: \quad \sum_i N_i(1 - E_i)(\lambda - v_u') + \sum_i \mu_i v_u' = 0 \tag{74}$$

$$T_f: \quad \lambda - v_f' - \nu = 0 \tag{75}$$

$$\begin{aligned} E_i: \quad & N_i(v_{i,r} - v_u) + N_i E_i(\overline{v_i'} - v_f')F_{ii} - \lambda N_i(T_i - T_u) \\ & + N_i \mu_i \left(\rho_i E_i \overline{v_i''} F_{ii} F_{ii} + \rho_i E_i \overline{v_i'} F_{iii} + v_{i,r}' F_{ii} + \rho_i v_{i,r}' F_{ii} + \int e_i'' v_i(\varphi) dG_i(\varphi)\right) = 0. \end{aligned} \tag{76}$$

$$\lambda: \quad R + \sum_i N_i(E_i T_i + (1 - E_i)T_u) + T_f = 0 \tag{77}$$

$$\mu_i: \quad \rho_i E_i \overline{v_i'} F_{ii} + v_{i,m} - v_u = 0 \tag{78}$$

$$\nu: \quad T_f^* - T_f = 0 \tag{79}$$

To derive the expression for the optimal participation tax, consider (76) and divide the expression by λ. Then, proceed as in the proof of the second part of Proposition 2 to arrive at

$$T_i - T_u = -\frac{1}{\lambda}(v_{i,r} - v_u) + \frac{w_i}{\varepsilon_{wi}}(b_f - b_i) - \frac{T_i}{\varepsilon_{Ti}}(1 - b_i). \tag{80}$$

Clearly, the only distinction with the result from Proposition 2 is that with restricted profit taxation, it is no longer true that $b_f = 1$ (in fact, one can directly see from (75) that whenever the constraint on profit taxation is binding, it must be that $b_f < 1$).

The proof of the second part is identical to the one from Proposition 3 (now using (73) instead of (42)) and hence omitted.

F Proof Proposition 6

The Lagrangean associated to the problem described by (40) reads

$$\begin{aligned} \mathcal{L} =& u(-T_u) + \kappa \left(F - \sum_i F_i N_i E_i - T_f + T_u\right) + \lambda(R + \sum_i N_i(E_i T_i + (1 - E_i)T_u) + T_f) \\ & + \sum_i \mu_i N_i \left(\rho_i \int_{\underline{\varphi}_i}^{\overline{\varphi}_i} e_i(E_i, \varphi) u_i'(F_i - T_i - \varphi) dG_i(\varphi) F_{ii} \right. \\ & \left. + \int_{\underline{\varphi}_i}^{\overline{\varphi}_i} e_i'(E_i, \varphi)(u_i(F_i - T_i - \varphi) - u(-T_u)) dG_i(\varphi)\right). \end{aligned} \tag{81}$$

Assuming independent labor markets, the first-order conditions are given by:

$$
\begin{aligned}
T_i: &\quad \lambda N_i E_i - \mu_i N_i \left(\rho_i E_i \overline{v_i''} F_{ii} + v'_{i,r} \right) = 0 && (82)\\
T_u: &\quad -v'_u + \kappa + \sum_i N_i(1-E_i)\lambda + \sum_i \mu_i N_i v'_u = 0 && (83)\\
T_f: &\quad -\kappa + \lambda = 0 && (84)\\
E_i: &\quad -\kappa F_{ii} N_i E_i + \lambda N_i (T_i - T_u) \\
&\quad + \mu_i N_i \left(\rho_i E_i \overline{v_i''} F_{ii} F_{ii} + \rho_i E_i \overline{v_i'} F_{iii} + v'_{i,r} F_{ii} + \rho_i v'_{i,r} F_{ii} + \int e_i'' v_i(\varphi) dG_i(\varphi) \right) = 0 && (85)\\
\kappa: &\quad \kappa(F - \sum_i F_i N_i E_i - T_f + T_u) = 0 && (86)\\
\lambda: &\quad R + \sum_i N_i (E_i T_i + (1-E_i) T_u) + T_f = 0 && (87)\\
\mu_i: &\quad \rho_i E_i \overline{v_i'} F_{ii} + v_{i,r} - v_u = 0. && (88)
\end{aligned}
$$

From the Kuhn-Tucker conditions, it must be that $\kappa \geq 0$. Suppose that $\kappa = 0$. In that case, (84) implies that $\lambda = 0$. Then, by (82) it must be that $\mu_i = 0$ for all i and for (83) to be satisfied, it is required that that $v'_u = 0$. This contradict the assumption that $u(\cdot)$ is a strictly increasing function. Hence, $\kappa = \lambda > 0$. Then, from (86) it must be that $v_u = v_f$, which confirms the first point stated in the Proposition. From (82) it then follows that $\mu_i > 0$ for all i. By reasoning analogous to that outlined in the proof of Proposition 3, it is then immediately implied that an increase in the degree of unionization is accompanied by a decrease in social welfare, as stated in the third point of the Proposition.

What remains to be shown, is that a Rawlsian government never wishes to subsidize participation on a net basis. This observation follows from (85), after using (82) to substitute out for μ_i. The sum of the first and third term is negative (by reasoning analogous to that in Appendix B), so that it must be that $T_i - T_u > 0$ for this expression to be satisfied. This completes the proof.